QUICK REVIEW 3.2

(For help, go to Section P.5.)

In Exercises 1 and 2, convert the percent to decimal form or the decimal into a percent.

1. 15%
2. 0.04
3. Show how to increase 23 by 7% using a single multiplication.
4. Show how to decrease 52 by 4% using a single multiplication.

In Exercises 5 and 6, solve the equation algebraically.

5. $40 \cdot b^2 = 160$
6. $243 \cdot b^3 = 9$

In Exercises 7–10, solve the equation numerically.

7. $782b^6 = 838$
8. $93b^5 = 521$
9. $672b^4 = 91$
10. $127b^7 = 56$

SECTION 3.2 EXERCISES

In Exercises 1–6, tell whether the function is an exponential growth function or exponential decay function, and find the constant percentage rate of growth or decay.

1. $P(t) = 3.5 \cdot 1.09^t$
2. $P(t) = 4.3 \cdot 1.018^t$
3. $f(x) = 78{,}963 \cdot 0.968^x$
4. $f(x) = 5607 \cdot 0.9968^x$
5. $g(t) = 247 \cdot 2^t$
6. $g(t) = 43 \cdot 0.05^t$

In Exercises 7–18, determine the exponential function that satisfies the given conditions.

7. Initial value = 5, increasing at a rate of 17% per year
8. Initial value = 52, increasing at a rate of 2.3% per day
9. Initial value = 16, decreasing at a rate of 50% per month
10. Initial value = 5, decreasing at a rate of 0.59% per week
11. Initial population = 28,900, decreasing at a rate of 2.6% per year
12. Initial population = 502,000, increasing at a rate of 1.7% per year
13. Initial height = 18 cm, growing at a rate of 5.2% per week
14. Initial mass = 15 g, decreasing at a rate of 4.6% per day
15. Initial mass = 0.6 g, doubling every 3 days
16. Initial population = 250, doubling every 7.5 hours
17. Initial mass = 592 g, halving once every 6 years
18. Initial mass = 17 g, halving once every 32 hours

In Exercises 19 and 20, determine a formula for the exponential function whose values are given in Table 3.11.

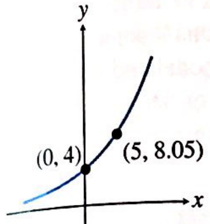
19. $f(x)$
20. $g(x)$

TABLE 3.11 VALUES FOR TWO EXPONENTIAL FUNCTIONS

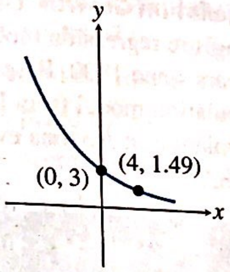
x	$f(x)$	$g(x)$
−2	1.472	−9.0625
−1	1.84	−7.25
0	2.3	−5.8
1	2.875	−4.64
2	3.59375	−3.712

In Exercises 21 and 22, determine a formula for the exponential function whose graph is shown in the figure.

21.



22.



In Exercises 23–26, find the logistic function that satisfies the given conditions.

23. Initial value = 10, limit to growth = 40, passing through (1, 20).

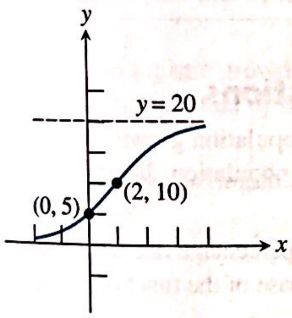
24. Initial value = 12, limit to growth = 60, passing through (1, 24).

25. Initial population = 16, maximum sustainable population = 128, passing through (5, 32).

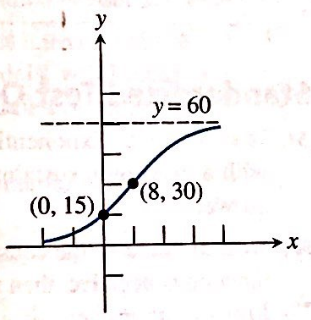
26. Initial height = 5, limit to growth = 30, passing through (3, 15).

In Exercises 27 and 28, determine a formula for the logistic function whose graph is shown in the figure.

27.



28.



29. **Exponential Growth** The 2000 population of Jacksonville, Florida was 736,000 and was increasing at the rate of 1.49% each year. At that rate, when will the population be 1 million?

30. **Exponential Growth** The 2000 population of Las Vegas, Nevada was 478,000 and is increasing at the rate of 6.28% each year. At that rate, when will the population be 1 million?

31. **Exponential Growth** The population of Smallville in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year.

(a) Estimate the population in 1915 and 1940.

(b) Predict when the population reached 50,000.

32. **Exponential Growth** The population of River City in the year 1910 was 4200. Assume the population increased at a rate of 2.25% per year.

(a) Estimate the population in 1930 and 1945.

(b) Predict when the population reached 20,000.

33. **Radioactive Decay** The half-life of a certain radioactive substance is 14 days. There are 6.6 g present initially.

(a) Express the amount of substance remaining as a function of time t.

(b) When will there be less than 1 g remaining?

34. **Radioactive Decay** The half-life of a certain radioactive substance is 65 days. There are 3.5 g present initially.

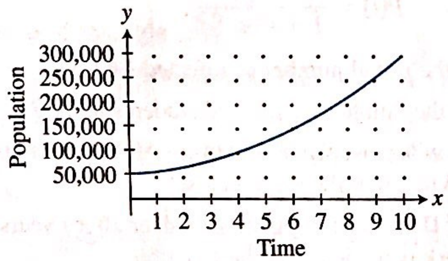
(a) Express the amount of substance remaining as a function of time t.

(b) When will there be less than 1 g remaining?

35. **Writing to Learn** Without using formulas or graphs, compare and contrast exponential functions and linear functions.

36. **Writing to Learn** Without using formulas or graphs, compare and contrast exponential functions and logistic functions.

37. **Writing to Learn** Using the population model that is graphed, explain why the time it takes the population to double (doubling time) is independent of the population size.



38. **Writing to Learn** Explain why the half-life of a radioactive substance is independent of the initial amount of the substance that is present.

39. **Bacteria Growth** The number B of bacteria in a petri dish culture after t hours is given by

$$B = 100e^{0.693t}.$$

When will the number of bacteria be 200? Estimate the doubling time of the bacteria.

40. **Radiocarbon Dating** The amount C in grams of carbon-14 present in a certain substance after t years is given by

$$C = 20e^{-0.0001216t}.$$

Estimate the half-life of carbon-14.

41. **Atmospheric Pressure** Determine the atmospheric pressure outside an aircraft flying at 52,800 ft (10 mi above sea level).

42. **Atmospheric Pressure** Find the altitude above sea level at which the atmospheric pressure is 2.5 lb/in.^{2}.

43. **Population Modeling** Use the 1950–1990 data in Table 3.12 and exponential regression to predict Los Angeles's population for 2000. Compare the result with the listed value for 2000.

44. Population Modeling Use the 1950–1990 data in Table 3.12 and exponential regression to predict Phoenix's population for 2000. Compare the result with the listed value for 2000.

TABLE 3.12 POPULATIONS OF TWO U.S. CITIES (IN THOUSANDS)

Year	Los Angeles	Phoenix
1950	1970	107
1960	2479	439
1970	2812	584
1980	2969	790
1990	3485	983
2000	3695	1,321

Source: U.S. Census Bureau.

45. Spread of Flu The number of students infected with flu at Springfield High School after t days is modeled by the function

$$P(t) = \frac{800}{1 + 49e^{-0.2t}}.$$

(a) What was the initial number of infected students?

(b) When will the number of infected students be 200?

(c) The school will close when 300 of the 800-student body are infected. When will the school close?

46. Population of Deer The population of deer after t years in Cedar State Park is modeled by the function

$$P(t) = \frac{1001}{1 + 90e^{-0.2t}}.$$

(a) What was the initial population of deer?

(b) When will the number of deer be 600?

(c) What is the maximum number of deer possible in the park?

47. Population Growth Using all of the data in Table 3.9, compute a logistic regression model, and use it to predict the U.S. population in 2010.

48. Population Growth Using the data in Table 3.13, confirm the model used in Example 8 of Section 3.1.

TABLE 3.13 POPULATION OF DALLAS, TEXAS

Year	Population
1950	434,462
1960	679,684
1970	844,401
1980	904,599
1990	1,006,877
2000	1,188,580

Source: U.S. Census Bureau.

49. Population Growth Using the data in Table 3.14, confirm the model used in Exercise 56 of Section 3.1.

50. Population Growth Using the data in Table 3.14, compute a logistic regression model for Arizona's population for t years since 1800. Based on your model and the New York population model from Exercise 56 of Section 3.1, will the population of Arizona ever surpass that of New York? If so, when?

TABLE 3.14 POPULATIONS OF TWO U.S. STATES (IN THOUSANDS)

Year	Arizona	New York
1900	0.1	7.3
1910	0.2	9.1
1920	0.3	10.3
1930	0.4	12.6
1940	0.5	13.5
1950	0.7	14.8
1960	1.3	16.8
1970	1.8	18.2
1980	2.7	17.6
1990	3.7	18.0
2000	5.1	19.0

Source: U.S. Census Bureau.

Standardized Test Questions

51. True or False Exponential population growth is constrained with a maximum sustainable population. Justify your answer.

52. True or False If the constant percentage rate of an exponential function is negative, then the base of the function is negative. Justify your answer.

In Exercises 53–56, you may use a graphing calculator to solve the problem.

53. Multiple Choice What is the constant percentage growth rate of $P(t) = 1.23 \cdot 1.049^t$?

(a) 49% **(b)** 23% **(c)** 4.9% **(d)** 2.3% **(e)** 1.23%

54. Multiple Choice What is the constant percentage decay rate of $P(t) = 22.7 \cdot 0.834^t$?

(a) 22.7% **(b)** 16.6% **(c)** 8.34%
(d) 2.27% **(e)** 0.834%

55. Multiple Choice A single cell amoeba doubles every 4 days. About how long will it take one amoeba to produce a population of 1000?

(a) 10 days **(b)** 20 days **(c)** 30 days
(d) 40 days **(e)** 50 days

56. **Multiple Choice** A rumor spreads logistically so that $S(t) = 789/(1 + 16 \cdot e^{-0.8t})$ models the number of persons who have heard the rumor by the end of t days. Based on this model, which of the following is **true**?

(a) After 0 days, 16 people have heard the rumor.

(b) After 2 days, 439 people have heard the rumor.

(c) After 4 days, 590 people have heard the rumor.

(d) After 6 days, 612 people have heard the rumor.

(e) After 8 days, 769 people have heard the rumor.

Explorations

57. **Population Growth** **(a)** Use the 1900–1990 data in Table 3.9 and *logistic* regression to predict the U.S. population for 2000.

(b) Writing to Learn Compare the prediction with the value listed in the table for 2000.

(c) Noting the results of Example 6, which model—exponential or logistic—makes the better prediction in this case?

58. **Population Growth** Use the data in Tables 3.9 and 3.15.

(a) Based on exponential growth models, will Mexico's population surpass that of the United States, and if so, when?

(b) Based on logistic growth models, will Mexico's population surpass that of the United States, and if so, when?

(c) What are the maximum sustainable populations for the two countries?

(d) Writing to Learn Which model—exponential or logistic—is more valid in this case? Justify your choice.

Table 3.15 Population of Mexico (in millions)

Year	Population
1900	13.6
1950	25.8
1960	34.9
1970	48.2
1980	66.8
1990	88.1
2001	101.9
2025	130.2
2050	154

Sources: 1992 Statesman's Yearbook and World Almanac and Book of Facts 2002.

Extending the Ideas

59. The **hyperbolic sine function** is defined by $\sinh(x) = (e^x - e^{-x})/2$. Prove that sinh is an odd function.

60. The **hyperbolic cosine function** is defined by $\cosh(x) = (e^x + e^{-x})/2$. Prove that cosh is an even function.

61. The **hyperbolic tangent function** is defined by $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$.

(a) Prove that $\tanh(x) = \sinh(x)/\cosh(x)$.

(b) Prove that tanh is an odd function.

(c) Prove that $f(x) = 1 + \tanh(x)$ is a logistic function.