SECTION 3.4 EXERCISES

In Exercises 1–12, assuming x and y are positive, use properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms.

1. $\ln 8x$
2. $\ln 9y$
3. $\log \frac{3}{x}$
4. $\log \frac{2}{y}$
5. $\log_2 y^5$
6. $\log_2 x^{-2}$
7. $\log x^3y^2$
8. $\log xy^3$
9. $\ln \frac{x^2}{y^3}$
10. $\log 1000x^4$
11. $\log \sqrt[4]{\frac{x}{y}}$
12. $\ln \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$

In Exercises 13–22, assuming x, y, and z are positive, use properties of logarithms to write the expression as a single logarithm.

13. $\log x + \log y$
14. $\log x + \log 5$
15. $\ln y - \ln 3$
16. $\ln x - \ln y$
17. $\frac{1}{3} \log x$
18. $\frac{1}{5} \log z$
19. $2 \ln x + 3 \ln y$
20. $4 \log y - \log z$
21. $4 \log (xy) - 3 \log (yz)$
22. $3 \ln (x^3y) + 2 \ln (yz^2)$

In Exercises 23–28, use the change-of-base formula and your calculator to evaluate the logarithm.

23. $\log_2 7$
24. $\log_5 19$
25. $\log_8 175$
26. $\log_{12} 259$
27. $\log_{0.5} 12$
28. $\log_{0.2} 29$

In Exercises 29–32, write the expression using only natural logarithms.

29. $\log_3 x$
30. $\log_7 x$
31. $\log_2 (a + b)$
32. $\log_5 (c - d)$

In Exercises 33–36, write the expression using only common logarithms.

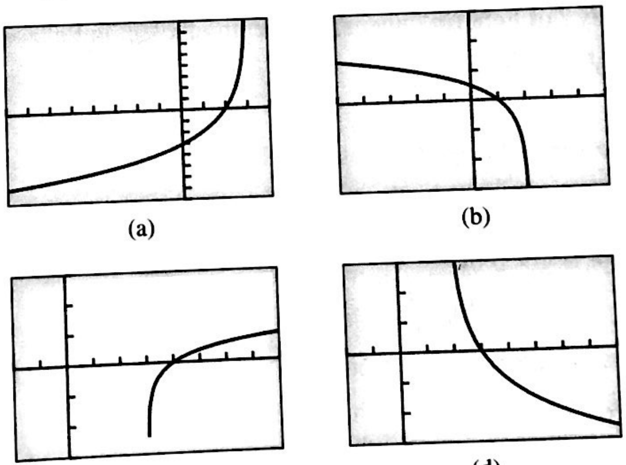
33. $\log_2 x$
34. $\log_4 x$
35. $\log_{1/2} (x + y)$
36. $\log_{1/3} (x - y)$
37. Prove the quotient rule of logarithms.
38. Prove the power rule of logarithms.

In Exercises 39–42, describe how to transform the graph of $g(x) = \ln x$ into the graph of the given function. Sketch the graph by hand and support with a grapher.

39. $f(x) = \log_4 x$
40. $f(x) = \log_7 x$
41. $f(x) = \log_{1/3} x$
42. $f(x) = \log_{1/5} x$

In Exercises 43–46, match the function with its graph. Identify the window dimensions, Xscl, and Yscl of the graph.

43. $f(x) = \log_4 (2 - x)$
44. $f(x) = \log_6 (x - 3)$
45. $f(x) = \log_{0.5} (x - 2)$
46. $f(x) = \log_{0.7} (3 - x)$



(a) (b) (c) (d)

In Exercises 47–50, graph the function, and analyze it for domain, range, continuity, increasing or decreasing behavior, asymptotes, and end behavior.

47. $f(x) = \log_2 (8x)$
48. $f(x) = \log_{1/3} (9x)$
49. $f(x) = \log (x^2)$
50. $f(x) = \ln (x^3)$

51. Sound Intensity Compute the sound intensity level in decibels for each sound listed in Table 3.21.

TABLE 3.21 Approximate Intensities for Selected Sounds

Sound	Intensity (Watts/m^2)
(a) Hearing threshold	10^{-12}
(b) Rustling leaves	10^{-11}
(c) Conversation	10^{-6}
(d) School cafeteria	10^{-4}
(e) Jack hammer	10^{-2}
(f) Pain threshold	1

Sources: J. J. Dwyer, College Physics (Belmont, CA: Wadsworth, 1984), and E. Connally et al., Functions Modeling Change (New York: Wiley, 2000).

52. Earthquake Intensity The **Richter scale** magnitude R of an earthquake is based on the features of the associated seismic wave and is measured by

$$R = \log (a/T) + B,$$

where a is the amplitude in μm (micrometers), T is the period in seconds, and B accounts for the weakening of the seismic wave due to the distance from the epicenter. Compute the earthquake magnitude R for each set of values.

(a) $a = 250$, $T = 2$, and $B = 4.25$

(b) $a = 300$, $T = 4$, and $B = 3.5$

53. Light Intensity in Lake Erie The relationship between intensity I of light (in lumens) at a depth of x feet in Lake Erie is given by

$$\log \frac{I}{12} = -0.00235x.$$

What is the intensity at a depth of 40 ft?

54. Light Intensity in Lake Superior The relationship between intensity I of light (in lumens) at a depth of x feet in Lake Superior is given by

$$\log \frac{I}{12} = -0.0125x.$$

What is the intensity at a depth of 10 ft?

55. Writing to Learn Use the change-of-base formula to explain how we know that the graph of $f(x) = \log_3 x$ can be obtained by applying a transformation to the graph of $g(x) = \ln x$.

56. Writing to Learn Use the change-of-base formula to explain how the graph of $f(x) = \log_{0.8} x$ can be obtained by applying transformations to the graph of $g(x) = \log x$.

Standardized Test Questions

57. True or False The logarithm of the product of two positive numbers is the sum of the logarithms of the numbers. Justify your answer.

58. True or False The logarithm of a positive number is positive. Justify your answer.

In Exercises 59–62, solve the problem without using a calculator.

59. Multiple Choice $\log 12 =$

(a) $3 \log 4$ **(b)** $\log 3 + \log 4$

(c) $4 \log 3$ **(d)** $\log 3 \cdot \log 4$

(e) $2 \log 6$

60. Multiple Choice $\log_9 64 =$

(a) $5 \log_3 2$ **(b)** $(\log_3 8)^2$

(c) $(\ln 64)/(\ln 9)$ **(d)** $2 \log_9 32$

(e) $(\log 64)/9$

61. Multiple Choice $\ln x^5 =$

(a) $5 \ln x$ **(b)** $2 \ln x^3$

(c) $x \ln 5$ **(d)** $3 \ln x^2$

(e) $\ln x^2 \cdot \ln x^3$

62. Multiple Choice $\log_{1/2} x^2 =$

(a) $-2 \log_2 x$ **(b)** $2 \log_2 x$

(c) $-0.5 \log_2 x$ **(d)** $0.5 \log_2 x$

(e) $-2 \log_2 |x|$

Explorations

63. (a) Compute the power regression model for the following data.

x	4	6.5	8.5	10
y	2816	31,908	122,019	275,000

(b) Predict the y value associated with $x = 7.1$ using the power regression model.

(c) Re-express the data in terms of their natural logarithms and make a scatter plot of $(\ln x, \ln y)$.

(d) Compute the linear regression model $(\ln y) = a(\ln x) + b$ for $(\ln x, \ln y)$.

(e) Confirm that $y = e^b \cdot x^a$ is the power regression model found in (a).

64. **(a)** Compute the power regression model for the following data.

x	2	3	4.8	7.7
y	7.48	7.14	6.81	6.41

(b) Predict the y value associated with $x = 9.2$ using the power regression model.

(c) Re-express the data in terms of their natural logarithms and make a scatter plot of $(\ln x, \ln y)$.

(d) Compute the linear regression model $(\ln y) = a(\ln x) + b$ for $(\ln x, \ln y)$.

(e) Confirm that $y = e^b \cdot x^a$ is the power regression model found in (a).

65. Keeping Warm—Revisited Recall from Exercise 55 of Section 2.2 that scientists have found the pulse rate r of mammals to be a power function of their body weight w.

(a) Re-express the data in Table 3.22 in terms of their *common* logarithms and make a scatter plot of $(\log w, \log r)$.

(b) Compute the linear regression model $(\log r) = a(\log w) + b$ for $(\log w, \log r)$.

(c) Superimpose the regression curve on the scatter plot.

(d) Use the regression equation to predict the pulse rate for a 450-kg horse. Is the result close to the 38 beats/min reported by A. J. Clark in 1927?

(e) **Writing to Learn** Why can we use either common or natural logarithms to re-express data that fit a power regression model?

Table 3.22 Weight and Pulse Rate of Selected Mammals

Mammal	Body weight (kg)	Pulse rate (beats/min)
Rat	0.2	420
Guinea pig	0.3	300
Rabbit	2	205
Small dog	5	120
Large dog	30	85
Sheep	50	70
Human	70	72

Source: A. J. Clark, Comparative Physiology of the Heart (New York: Macmillan, 1927).

66. Let $a = \log 2$ and $b = \log 3$. Then, for example $\log 6 = a + b$. List the common logs of all the positive integers less than 100 that can be expressed in terms of a and b, writing equations such as $\log 6 = a + b$ for each case.

Extending the Ideas

67. Solve $\ln x > \sqrt[3]{x}$.

68. Solve $1.2^x \le \log_{1.2} x$.

69. Group Activity Work in groups of three. Have each group member graph and compare the domains for one pair of functions.

(a) $f(x) = 2\ln x + \ln(x - 3)$ and $g(x) = \ln x^2(x - 3)$

(b) $f(x) = \ln(x + 5) - \ln(x - 5)$ and $g(x) = \ln \dfrac{x + 5}{x - 5}$

(c) $f(x) = \log(x + 3)^2$ and $g(x) = 2\log(x + 3)$

Writing to Learn After discussing your findings, write a brief group report that includes your overall conclusions and insights.

70. Prove the change-of-base formula for logarithms.

71. Prove that $f(x) = \log x/\ln x$ is a constant function with restricted domain by finding the exact value of the constant $\log x/\ln x$ expressed as a common logarithm.

72. Graph $f(x) = \ln(\ln(x))$, and analyze it for domain, range, continuity, increasing or decreasing behavior, symmetry, asymptotes, end behavior, and invertibility.