CHAPTER 3 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

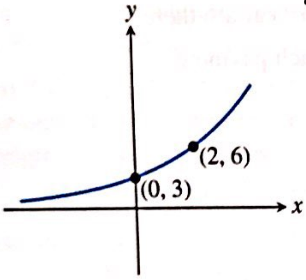
In Exercises 1 and 2, compute the exact value of the function for the given x value without using a calculator.

1. $f(x) = -3 \cdot 4^x$ for $x = \frac{1}{3}$

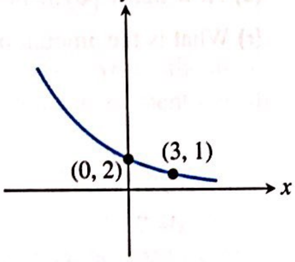
2. $f(x) = 6 \cdot 3^x$ for $x = -\frac{3}{2}$

In Exercises 3 and 4, determine a formula for the exponential function whose graph is shown in the figure.

3.



4.



In Exercises 5–10, describe how to transform the graph of f into the graph of $g(x) = 2^x$ or $h(x) = e^x$. Sketch the graph by hand and support your answer with a grapher.

5. $f(x) = 4^{-x} + 3$

6. $f(x) = -4^{-x}$

7. $f(x) = -8^{-x} - 3$

8. $f(x) = 8^{-x} + 3$

9. $f(x) = e^{2x-3}$

10. $f(x) = e^{3x-4}$

In Exercises 11 and 12, find the y-intercept and the horizontal asymptotes.

11. $f(x) = \dfrac{100}{5 + 3e^{-0.05x}}$

12. $f(x) = \dfrac{50}{5 + 2e^{-0.04x}}$

In Exercises 13 and 14, state whether the function is an exponential growth function or an exponential decay function, and describe its end behavior using limits.

13. $f(x) = e^{4-x} + 2$

14. $f(x) = 2(5^{x-3}) + 1$

In Exercises 15–18, graph the function, and analyze it for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.

15. $f(x) = e^{3-x} + 1$

16. $g(x) = 3(4^{x+1}) - 2$

17. $f(x) = \dfrac{6}{1 + 3 \cdot 0.4^x}$

18. $g(x) = \dfrac{100}{4 + 2e^{-0.01x}}$

In Exercises 19–22, find the exponential function that satisfies the given conditions.

19. Initial value = 24, increasing at a rate of 5.3% per day

20. Initial population = 67,000, increasing at a rate of 1.67% per year

21. Initial height = 18 cm, doubling every 3 weeks

22. Initial mass = 117 g, halving once every 262 hours

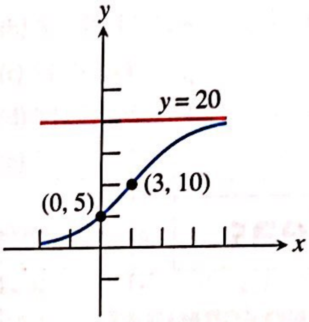
In Exercises 23 and 24, find the logistic function that satisfies the given conditions.

23. Initial value = 12, limit to growth = 30, passing through (2, 20).

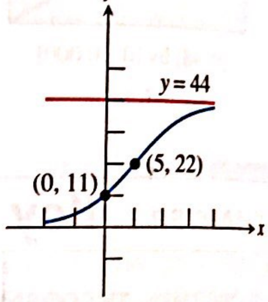
24. Initial height = 6, limit to growth = 20, passing through (3, 15).

In Exercises 25 and 26, determine a formula for the logistic function whose graph is shown in the figure.

25.



26.



In Exercises 27–30, evaluate the logarithmic expression without using a calculator.

27. $\log_2 32$

28. $\log_3 81$

29. $\log \sqrt[3]{10}$

30. $\ln \dfrac{1}{\sqrt{e^7}}$

In Exercises 31–34, rewrite the equation in exponential form.

31. $\log_3 x = 5$

32. $\log_2 x = y$

33. $\ln \dfrac{x}{y} = -2$

34. $\log \dfrac{a}{b} = -3$

In Exercises 35–38, describe how to transform the graph of $y = \log_2 x$ into the graph of the given function. Sketch the graph by hand and support with a grapher.

35. $f(x) = \log_2 (x + 4)$

36. $g(x) = \log_2 (4 - x)$

37. $h(x) = -\log_2 (x - 1) + 2$

38. $h(x) = -\log_2 (x + 1) + 4$

In Exercises 39–42, graph the function, and analyze it for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.

39. $f(x) = x \ln x$

40. $f(x) = x^2 \ln x$

41. $f(x) = x^2 \ln |x|$

42. $f(x) = \dfrac{\ln x}{x}$

In Exercises 43–54, solve the equation.

43. $10^x = 4$

44. $e^x = 0.25$

45. $1.05^x = 3$

46. $\ln x = 5.4$

47. $\log x = -7$

48. $3^{x-3} = 5$

49. $3 \log_2 x + 1 = 7$

50. $2 \log_3 x - 3 = 4$

51. $\dfrac{3^x - 3^{-x}}{2} = 5$

52. $\dfrac{50}{4 + e^{2x}} = 11$

53. $\log (x + 2) + \log (x - 1) = 4$

54. $\ln (3x + 4) - \ln (2x + 1) = 5$

In Exercises 55 and 56, write the expression using only natural logarithms.

55. $\log_2 x$

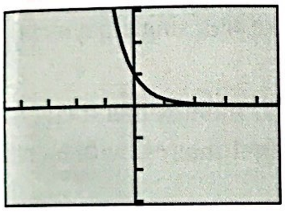
56. $\log_{1/6} (6x^2)$

In Exercises 57 and 58, write the expression using only common logarithms.

57. $\log_5 x$

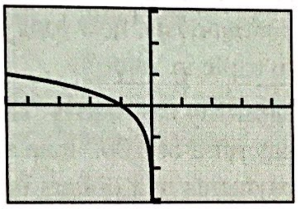
58. $\log_{1/2} (4x^3)$

In Exercises 59–62, match the function with its graph. All graphs are drawn in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.



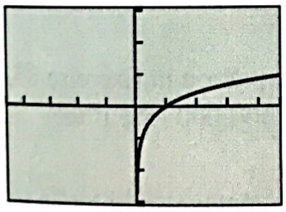
[–4.7, 4.7] by [–3.1, 3.1]

(a)



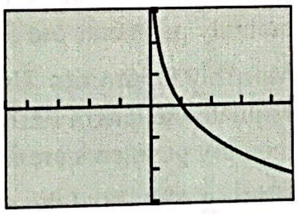
[–4.7, 4.7] by [–3.1, 3.1]

(b)



[–4.7, 4.7] by [–3.1, 3.1]

(c)



[–4.7, 4.7] by [–3.1, 3.1]

(d)

59. $f(x) = \log_5 x$

60. $f(x) = \log_{0.5} x$

61. $f(x) = \log_5 (-x)$

62. $f(x) = 5^{-x}$

63. **Compound Interest** Find the amount A accumulated after investing a principal $P = \$450$ for 3 years at an interest rate of 4.6% compounded annually.

64. **Compound Interest** Find the amount A accumulated after investing a principal $P = \$4800$ for 17 years at an interest rate 6.2% compounded quarterly.

65. **Compound Interest** Find the amount A accumulated after investing a principal P for t years at interest rate r compounded continuously.

66. **Future Value** Find the future value FV accumulated in an annuity after investing periodic payments R for t years at an annual interest rate r, with payments made and interest credited k times per year.

67. **Present Value** Find the present value PV of a loan with an annual interest rate $r = 5.5\%$ and periodic payments $R = \$550$ for a term of $t = 5$ years, with payments made and interest charged 12 times per year.

68. **Present Value** Find the present value PV of a loan with an annual interest rate $r = 7.25\%$ and periodic payments $R = \$953$ for a term of $t = 15$ years, with payments mac and interest charged 26 times per year.

In Exercises 69 and 70, determine the value of k so that the graph of f passes through the given point.

69. $f(x) = 20e^{-kx}$, $(3, 50)$

70. $f(x) = 20e^{-kx}$, $(1, 30)$

In Exercises 71 and 72, use the data in Table 3.28.

Table 3.28 Populations of Two U.S. States (in millions)

Year	Georgia	Illinois
1900	2.2	4.8
1910	2.6	5.6
1920	2.9	6.5
1930	2.9	7.6
1940	3.1	7.9
1950	3.4	8.7
1960	3.9	10.1
1970	4.6	11.1
1980	5.5	11.4
1990	6.5	11.4
2000	8.2	12.4

Source: U.S. Census Bureau as reported in 1999 New York Times Almanac.

71. **Modeling Population** Find an exponential regression model for Georgia's population, and use it to predict tl population in 2005.

72. **Modeling Population** Find a logistic regression moc for Illinois' population, and use it to predict the popul in 2010.

73. **Drug Absorption** A drug is administered intravenou for pain. The function $f(t) = 90 - 52 \ln (1 + t)$, whe $0 \le t \le 4$, gives the amount of the drug in the body ; t hours.

(a) What was the initial $(t = 0)$ number of units of di administered?

(b) How much is present after 2 h?

(c) Draw the graph of f.

74. **Population Decrease** The population of Metroville is 123,000 and is decreasing by 2.4% each year.

(a) Write a function that models the population as a function of time t.

(b) Predict when the population will be 90,000.

75. **Population Decrease** The population of Preston is 89,000 and is decreasing by 1.8% each year.

(a) Write a function that models the population as a function of time t.

(b) Predict when the population will be 50,000.

76. **Spread of Flu** The number P of students infected with flu at Northridge High School t days after exposure is modeled by

$$P(t) = \frac{300}{1 + e^{4-t}}.$$

(a) What was the initial ($t = 0$) number of students infected with the flu?

(b) How many students were infected after 3 days?

(c) When will 100 students be infected?

(d) What would be the maximum number of students infected?

77. **Rabbit Population** The number of rabbits in Elkgrove doubles every month. There are 20 rabbits present initially.

(a) Express the number of rabbits as a function of the time t.

(b) How many rabbits were present after 1 year? after 5 years?

(c) When will there be 10,000 rabbits?

78. **Guppy Population** The number of guppies in Susan's aquarium doubles every day. There are four guppies initially.

(a) Express the number of guppies as a function of time t.

(b) How many guppies were present after 4 days? after 1 week?

(c) When will there be 2000 guppies?

79. **Radioactive Decay** The half-life of a certain radioactive substance is 1.5 sec. The initial amount of substance is S_0 grams.

(a) Express the amount of substance S remaining as a function of time t.

(b) How much of the substance is left after 1.5 sec? after 3 sec?

(c) Determine S_0 if there was 1 g left after 1 min.

80. **Radioactive Decay** The half-life of a certain radioactive substance is 2.5 sec. The initial amount of substance is S_0 grams.

(a) Express the amount of substance S remaining as a function of time t.

(b) How much of the substance is left after 2.5 sec? after 7.5 sec?

(c) Determine S_0 if there was 1 g left after 1 min.

81. **Richter Scale** Afghanistan suffered two major earthquakes in 1998. The one on February 4 had a Richter magnitude of 6.1, causing about 2300 deaths, and the one on May 30 measured 6.9 on the Richter scale, killing about 4700 people. How many times more powerful was the deadlier quake?

82. **Chemical Acidity** The pH of seawater is 7.6, and the pH of milk of magnesia is 10.5.

(a) What are their hydrogen-ion concentrations?

(b) How many times greater is the hydrogen-ion concentration of the seawater than that of milk of magnesia?

(c) By how many orders of magnitude do the concentrations differ?

83. **Annuity Finding Time** If Joenita invests $1500 into a retirement account with an 8% interest rate compounded quarterly, how long will it take this single payment to grow to $3750?

84. **Annuity Finding Time** If Juan invests $12,500 into a retirement account with a 9% interest rate compounded continuously, how long will it take this single payment to triple in value?

85. **Monthly Payments** The time t in months that it takes to pay off a $60,000 loan at 9% annual interest with monthly payments of x dollars is given by

$$t = 133.83 \ln\left(\frac{x}{x - 450}\right).$$

Estimate the length (term) of the $60,000 loan if the monthly payments are $700.

86. **Monthly Payments** Using the equation in Exercise 85, estimate the length (term) of the $60,000 loan if the monthly payments are $500.

87. **Finding APY** Find the annual percentage yield for an investment with an interest rate of 8.25% compounded monthly.

88. **Finding APY** Find the annual percentage yield that can be used to advertise an account that pays interest at 7.20% compounded countinuously.

89. **Light Absorption** The Beer-Lambert law of absorption applied to Lake Superior states that the light intensity I (in lumens) at a depth of x feet satisfies the equation

$$\log \frac{I}{12} = -0.0125x.$$

Find the light intensity at a depth of 25 ft.

90. For what values of b is $\log_b x$ a vertical stretch of $y = \ln x$? A vertical shrink of $y = \ln x$?

91. For what values of b is $\log_b x$ a vertical stretch of $y = \log x$? A vertical shrink of $y = \log x$?

92. If $f(x) = ab^x$, $a > 0$, $b > 0$, prove that $g(x) = \ln f(x)$ is a linear function. Find its slope and y-intercept.

93. **Spread of Flu** The number of students infected with flu after t days at Springfield High School is modeled by the function

$$P(t) = \frac{1600}{1 + 99e^{-0.4t}}.$$

(a) What was the initial number of infected students?

(b) When will 800 students be infected?

(c) The school will close when 400 of the 1600 student body are infected. When would the school close?

94. **Population of Deer** The population P of deer after t years in Briggs State Park is modeled by the function

$$P(t) = \frac{1200}{1 + 99e^{-0.4t}}.$$

(a) What was the inital population of deer?

(b) When will there be 1000 deer?

(c) What is the maximum number of deer planned for the park?

95. **Newton's Law of Cooling** A cup of coffee cooled from 96°C to 65°C after 8 min in a room at 20°C. When will it cool to 25°C?

96. **Newton's Law of Cooling** A cake is removed from an oven at 220° F and cools to 150°F after 35 min in a room at 75°F. When will it cool to 95°F?

97. The function

$$f(x) = 100\frac{(1 + 0.09/4)^x - 1}{0.09/4}$$

describes the future value of a certain annuity.

(a) What is the annual interest rate?

(b) How many payments per year are there?

(c) What is the amount of each payment?

98. The function

$$g(x) = 200\frac{1 - (1 + 0.11/4)^{-x}}{0.11/4}$$

describes the present value of a certain annuity.

(a) What is the annual interest rate?

(b) How many payments per year are there?

(c) What is the amount of each payment?

99. **Simple Interest versus Compounding Continuously** Grace purchases a $1000 certificate of deposit that will earn 5% each year. The interest will be mailed to her, so she will not earn interest on her interest.

(a) Show that after t years, the total amount of interest she receives from her investment plus the original $1000 is given by

$$f(t) = 1000(1 + 0.05t).$$

(b) Grace invests another $1000 at 5% compounded continuously. Make a table that compares the values of the two investments for $t = 1, 2, \ldots, 10$ years.