Day 6: Polar Coordinates

You know two ways to graph so far…

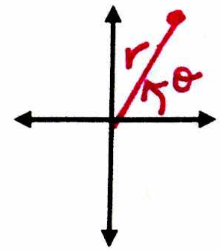
- Rectangular Plane (x, y)
- Complex Plane (a, b)

Now we are going to graph **POLAR COORDINATES!**

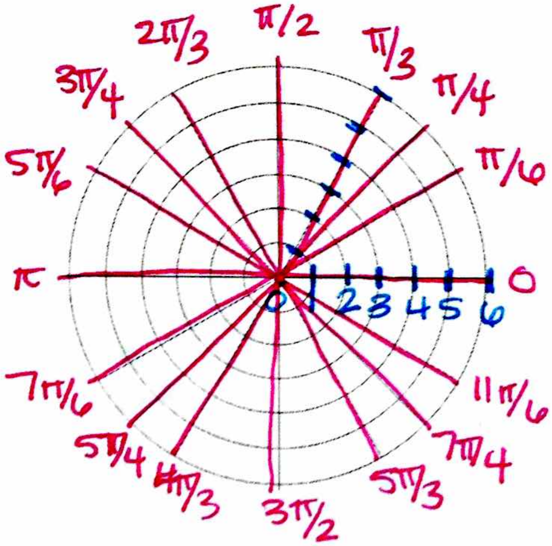
(r, θ)

Where r is radius

And θ is angle



The Polar Graph:



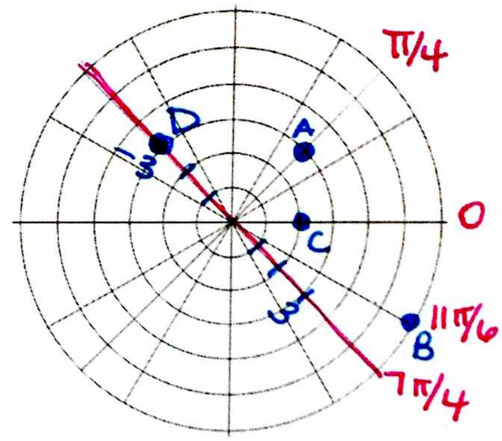
Example 1: Plot each point on the graph.

a) $(3, \pi/4)$

b) $(6, 11\pi/6)$

c) $(2, 0)$

d) $(-3, 7\pi/4)$



Rectangular coordinates (x, y) have only one unique representation, while polar coordinates can be written multiple ways.

Example 2: Rewrite the point in three different ways.

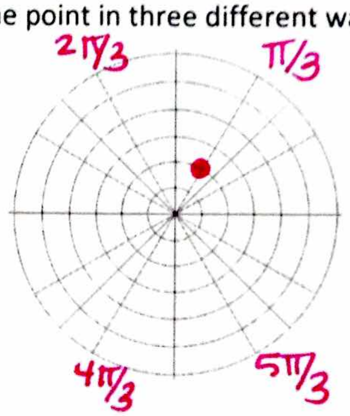
a) $(2, \frac{7\pi}{3})$ → π/3

(−2, 4π/3)

(2, −5π/3)

(−2, −2π/3)

(2, π/3)

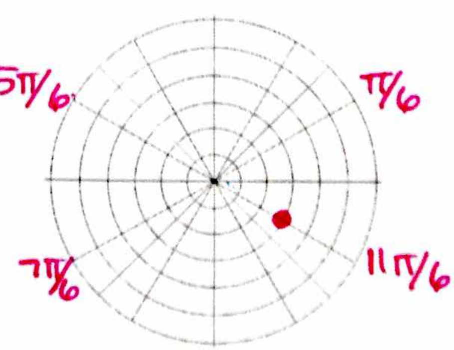


b) $(3, \frac{11\pi}{6})$

(−3, 5π/6)

(3, −π/6)

(−3, −7π/6)

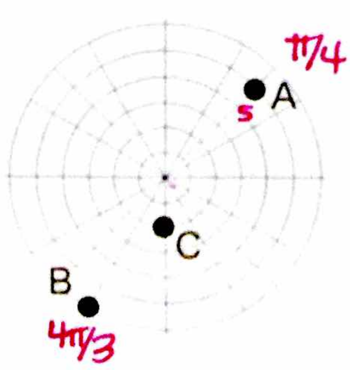


Example 3: Determine the polar coordinate of each of the points on the graph.

A (5, π/4)

B (6, 4π/3)

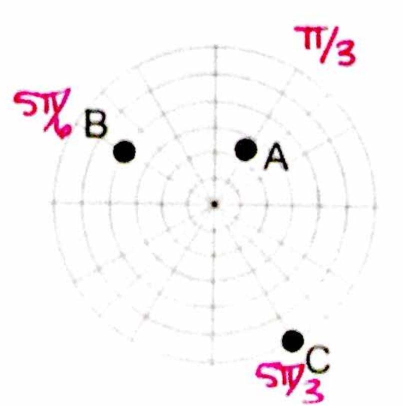
C (2, 3π/2)



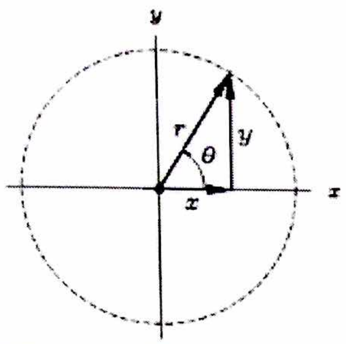
A (2.5, π/3)

B (4, 5π/6)

C (6, 5π/3)



Converting between Rectangular & Polar Coordinates:



$r^2 = x^2+y^2$ $r = \sqrt{x^2+y^2}$

$\cos\theta = \frac{x}{r}$ $x = r\cos\theta$

$\sin\theta = \frac{y}{r}$ $y = r\sin\theta$

$\tan\theta = \frac{y}{x}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Example 4: Convert to rectangular coordinates.

$(r,\theta) \to (x,y)$

a) $(2,\pi)$

$x = 2\cos(\pi)$ $y = 2\sin(\pi)$

$x = 2(-1)$ $y = 2(0)$

$x = -2$ $y = 0$

$(-2, 0)$

b) $(\sqrt{3}, \frac{\pi}{6})$

$(\sqrt{3}/2, 1/2)$

$x = \sqrt{3}\cos(\pi/6)$ $y = \sqrt{3}\sin(\pi/6)$

$x = \sqrt{3}(\sqrt{3}/2)$ $y = \sqrt{3}(1/2)$

$x = 3/2$ $y = \sqrt{3}/2$

$(3/2, \sqrt{3}/2)$

c) $(3, \frac{5\pi}{3})$

$(1/2, -\sqrt{3}/2)$

$x = 3\cos(5\pi/3)$ $y = 3\sin(5\pi/3)$

$x = 3(1/2)$ $y = 3(-\sqrt{3}/2)$

$x = 3/2$ $y = -3\sqrt{3}/2$

$(3/2, -3\sqrt{3}/2)$

d) $(-3, \frac{3\pi}{2})$

$x = -3\cos(3\pi/2)$ $y = -3\sin(3\pi/2)$

$x = -3(0)$ $y = -3(-1)$

$x = 0$ $y = 3$

$(0, 3)$

Example 5: Convert to polar coordinates.

$(x,y) \to (r,\theta)$

a) $(-1, 1)$

$3\pi/4$ 1 -1 $-45°$

$r = \sqrt{(-1)^2+(1)^2}$ $\theta = \tan^{-1}\left(\frac{1}{-1}\right)$

$r = \sqrt{1+1}$ $\theta = \tan^{-1}(-1)$

$r = \sqrt{2}$ $\theta = -45° + 180°$

$(\sqrt{2}, 3\pi/4)$ $\theta = 135°$ or $3\pi/4$

b) $(0, 2)$

$\pi/2$

$r = \sqrt{0^2+2^2}$ $\theta = \tan^{-1}\left(\frac{2}{0}\right)$

$r = \sqrt{4}$ $\theta = \tan^{-1}(\text{und.})$

$r = 2$ $\theta = \pi/2$ or $90°$

$(2, \pi/2)$

c) $(\sqrt{2}, \sqrt{2})$

$\pi/4$ $\sqrt{2}$ $\sqrt{2}$

$r = \sqrt{(\sqrt{2})^2+(\sqrt{2})^2}$

$r = \sqrt{2+2}$ $\theta = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$r = \sqrt{4}$ $\theta = \tan^{-1}(1)$

$r = 2$ $\theta = \pi/4$ or $45°$

$(2, \pi/4)$

d) $(-8, -2)$

-2 $14.04°$ -8 θ

$r = \sqrt{(-8)^2+(-2)^2}$

$r = \sqrt{64+4}$ $\theta = \tan^{-1}\left(\frac{-2}{-8}\right)$

$r = \sqrt{68}$ $\theta = \tan^{-1}\left(\frac{1}{4}\right)$

$r = 2\sqrt{17}$ $\theta = 14.04° + 180°$

$\theta = 194.04°$

$(2\sqrt{17}, 194.04°)$

Converting Equations from Rectangular Form to Polar Form: $x = r\cos\theta \quad y = r\sin\theta$

$r^2 = x^2 + y^2$

- All polar equations are written as $r =$ and must contain only r and θ
- When solving polar equations you can divide by r since r represents a radius and a radius cannot be 0 you can divide.

Example 6: Convert the equation from rectangular form to polar form. x & $y \rightarrow r$ & θ

a) $y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$\frac{\cancel{r}\sin\theta}{\cancel{r}} = \frac{r^{\cancel{2}}\cos^2\theta}{\cancel{r}}$

$\frac{\sin\theta}{\cos^2\theta} = \frac{r\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}$

$\frac{\sin\theta \cdot 1}{\cos\theta \cdot \cos\theta} = r$

$\boxed{\tan\theta\sec\theta = r}$

b) $y = 4$

$\frac{r\sin\theta}{\sin\theta} = \frac{4}{\sin\theta}$

$\boxed{r = \frac{4}{\sin\theta}}$

c) $x^2 + y^2 = 9$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 9$

$r^2\cos^2\theta + r^2\sin^2\theta = 9$

$r^2(\cos^2\theta + \sin^2\theta) = 9$

$r^2(1) = 9$

$r^2 = 9$

$\boxed{r = \pm 3}$

d) $3x - 2y = 6$

$3r\cos\theta - 2r\sin\theta = 6$

$r(3\cos\theta - 2\sin\theta) = 6$

$\boxed{r = \frac{6}{3\cos\theta - 2\sin\theta}}$

e) $2x + y = 5$

$2r\cos\theta + r\sin\theta = 5$

$r(2\cos\theta + \sin\theta) = 5$

$\boxed{r = \frac{5}{2\cos\theta + \sin\theta}}$

Example 7: Convert the equation from polar form to rectangular form. r & $\theta \rightarrow x$ & y

a) $(r)^2 = (2)^2$

$r^2 = 4$

$\boxed{x^2 + y^2 = 4}$

circle, w/radius 2 @ (0,0)

b) $\theta = \frac{\pi}{3}$

$\frac{1}{2}, \frac{\sqrt{3}}{2}$

* take tan of both sides

$\tan(\theta) = \tan(\pi/3)$

$\frac{y}{x} = \sqrt{3}$

$\boxed{y = \sqrt{3}x}$

line w/slope of $\sqrt{3}$

c) $r = \sec\theta$

$r = \frac{1}{\cos\theta}$

$r\cos\theta = 1$

$\boxed{x = 1}$

vertical line

d) $r(r) = (5\cos\theta)r$

complete the square *

$r^2 = 5r\cos\theta$

$x^2 + y^2 = 5x$

$(x^2 - 5x + \frac{25}{4}) + y^2 = 0 + \frac{25}{4}$

$\boxed{(x - \frac{5}{2})^2 + y^2 = \frac{25}{4}}$

circle w/radius of $\frac{5}{2} = 2.5$ @ (2.5, 0)

e) $r = \frac{3}{4\cos\theta - \sin\theta}$

$r(4\cos\theta - \sin\theta) = 3$

$4r\cos\theta - r\sin\theta = 3$

$4x - y = 3$

$-y = -4x + 3$

$\boxed{y = 4x - 3}$

line!