QUICK REVIEW 3.1

(For help, go to Sections A.1 and P.1.)

In Exercises 1–4, evaluate the expression without using a calculator.

1. $\sqrt[3]{-216}$
2. $\sqrt[3]{\frac{125}{8}}$
3. $27^{2/3}$
4. $4^{5/2}$

In Exercises 5–8, rewrite the expression using a single positive exponent.

5. $(2^{-3})^4$
6. $(3^4)^{-2}$
7. $(a^{-2})^3$
8. $(b^{-3})^{-5}$

In Exercises 9–10, use a calculator to evaluate the expression.

9. $\sqrt[5]{-5.37824}$
10. $\sqrt[4]{92.3521}$

SECTION 3.1 EXERCISES

In Exercises 1–6, which of the following are exponential functions? For those that are exponential functions, state the initial value and the base. For those that are not, explain why not.

1. $y = x^8$
2. $y = 3^x$
3. $y = 5^x$
4. $y = 4^2$
5. $y = x^{\sqrt{x}}$
6. $y = x^{1.3}$

In Exercises 7–10, compute the exact value of the function for the given x-value without using a calculator.

7. $f(x) = 3 \cdot 5^x$ for $x = 0$
8. $f(x) = 6 \cdot 3^x$ for $x = -2$
9. $f(x) = -2 \cdot 3^x$ for $x = 1/3$
10. $f(x) = 8 \cdot 4^x$ for $x = -3/2$

In Exercises 11 and 12, determine a formula for the exponential function whose values are given in Table 3.6.

11. $f(x)$

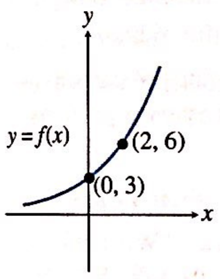
12. $g(x)$

Table 3.6 Values for Two Exponential Functions

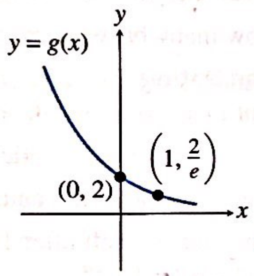
x	$f(x)$	$g(x)$
-2	6	108
-1	3	36
0	3/2	12
1	3/4	4
2	3/8	4/3

In Exercises 13 and 14, determine a formula for the exponential function whose graph is shown in the figure.

13. $f(x)$



14. $g(x)$



In Exercises 15–24, describe how to transform the graph of f into the graph of g. Sketch the graph by hand and support your answer with a grapher.

15. $f(x) = 2^x,\ g(x) = 2^{x-3}$

16. $f(x) = 3^x,\ g(x) = 3^{x+4}$

17. $f(x) = 4^x,\ g(x) = 4^{-x}$

18. $f(x) = 2^x,\ g(x) = 2^{5-x}$

19. $f(x) = 0.5^x,\ g(x) = 3 \cdot 0.5^x + 4$

20. $f(x) = 0.6^x,\ g(x) = 2 \cdot 0.6^{3x}$

21. $f(x) = e^x,\ g(x) = e^{-2x}$

22. $f(x) = e^x,\ g(x) = -e^{-3x}$

23. $f(x) = e^x,\ g(x) = 2e^{3-3x}$

24. $f(x) = e^x,\ g(x) = 3e^{2x} - 1$

In Exercises 25–30, **(a)** match the given function with its graph. **(b) Writing to Learn** Explain how to make the choice without using a grapher.

25. $y = 3^x$

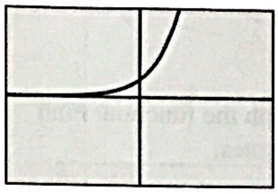
26. $y = 2^{-x}$

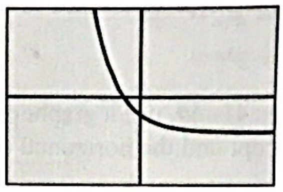
27. $y = -2^x$

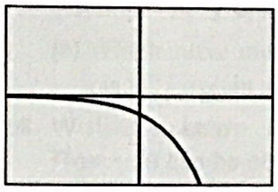
28. $y = -0.5^x$

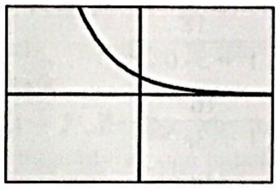
29. $y = 3^{-x} - 2$

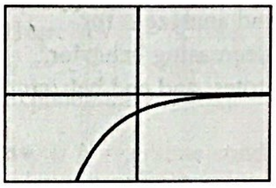
30. $y = 1.5^x - 2$

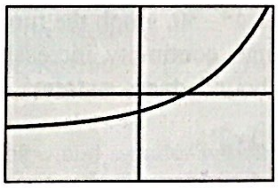

(a)


(b)


(c)


(d)


(e)


(f)

In Exercises 31–34, state whether the function is an exponential growth function or exponential decay function, and describe its end behavior using limits.

31. $f(x) = 3^{-2x}$

32. $f(x) = \left(\frac{1}{e}\right)^x$

33. $f(x) = 0.5^x$

34. $f(x) = 0.75^{-x}$

In Exercises 35–38, solve the inequality graphically.

35. $9^x < 4^x$

36. $6^{-x} > 8^{-x}$

37. $\left(\frac{1}{4}\right)^x > \left(\frac{1}{3}\right)^x$

38. $\left(\frac{1}{3}\right)^x < \left(\frac{1}{2}\right)^x$

In Exercises 39 and 40, use the properties of exponents to prove that two of the given three exponential functions are identical. Support graphically.

39. Group Activity

(a) $y_1 = 3^{2x+4}$

(b) $y_2 = 3^{2x} + 4$

(c) $y_3 = 9^{x+2}$

40. Group Activity

(a) $y_1 = 4^{3x-2}$

(b) $y_2 = 2(2^{3x-2})$

(c) $y_3 = 2^{3x-1}$

In Exercises 41–44, use a grapher to graph the function. Find the y-intercept and the horizontal asymptotes.

41. $f(x) = \dfrac{12}{1 + 2 \cdot 0.8^x}$

42. $f(x) = \dfrac{18}{1 + 5 \cdot 0.2^x}$

43. $f(x) = \dfrac{16}{1 + 3e^{-2x}}$

44. $g(x) = \dfrac{9}{1 + 2e^{-x}}$

In Exercises 45–50, graph the function and analyze it for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.

45. $f(x) = 3 \cdot 2^x$

46. $f(x) = 4 \cdot 0.5^x$

47. $f(x) = 4 \cdot e^{3x}$

48. $f(x) = 5 \cdot e^{-x}$

49. $f(x) = \dfrac{5}{1 + 4 \cdot e^{-2x}}$

50. $f(x) = \dfrac{6}{1 + 2 \cdot e^{-x}}$

51. Population Growth Using the data in Table 3.7 and assuming the growth is exponential, when will the population of Austin surpass 800,000 persons?

52. Population Growth Using the data in Table 3.7 and assuming the growth is exponential, when will the population of Columbus surpass 800,000 persons?

53. Population Growth Using the data in Table 3.7 and assuming the growth is exponential, when will the populations of Austin and Columbus be equal?

54. Population Growth Using the data in Table 3.7 and assuming the growth is exponential, which city—Austin or Columbus—will reach a population of 1 million first, and in what year?

TABLE 3.7 POPULATIONS OF TWO MAJOR U.S. CITIES

City	1990 Population	2000 Population
Austin, Texas	465,622	656,562
Columbus, Ohio	632,910	711,470

Source: World Almanac and Book of Facts 2002.

55. Population Growth Using 20th-century U.S. census data, the population of Ohio can be modeled by $P(t) = 12.79/(1 + 2.402e^{-0.0309x})$, where P is the population in millions and t is the number of years since 1900. Based on this model, when was the population of Ohio 10 million?

56. Population Growth Using 20th century U.S. census data, the population of New York state can be modeled by

$$P(t) = \frac{19.875}{1 + 57.993e^{-0.035005t}},$$

where P is the population in millions and t is the number of years since 1800. Based on this model,

(a) What was the population of New York in 1850?

(b) What will New York state's population be in 2010?

(c) What is New York's *maximum sustainable population* (limit to growth)?

57. Bacteria Growth The number B of bacteria in a petri dish culture after t hours is given by

$$B = 100e^{0.693t}.$$

(a) What was the initial number of bacteria present?

(b) How many bacteria are present after 6 hours?

58. Carbon Dating The amount C in grams of carbon-14 present in a certain substance after t years is given by

$$C = 20e^{-0.0001216t}.$$

(a) What was the initial amount of carbon-14 present?

(b) How much is left after 10,400 years? When will the amount left be 10 g?

Standardized Test Questions

59. True or False Every exponential function is strictly increasing. Justify your answer.

60. True or False Every logistic growth function has two horizontal asymptotes. Justify your answer.

In Exercises 61–64, solve the problem without using a calculator.

61. Multiple Choice Which of the following functions is exponential?

(a) $f(x) = a^2$

(b) $f(x) = x^3$

(c) $f(x) = x^{2/3}$

(d) $f(x) = \sqrt[3]{x}$

(e) $f(x) = 8^x$

62. Multiple Choice What point do all functions of the form $f(x) = b^x$ $(b > 0)$ have in common?

(a) $(1, 1)$
(b) $(1, 0)$
(c) $(0, 1)$
(d) $(0, 0)$
(e) $(-1, -1)$

63. Multiple Choice The growth factor for $f(x) = 4 \cdot 3^x$ is

(a) 3. **(b)** 4. **(c)** 12.
(d) 64. **(e)** 81.

64. Multiple Choice For $x > 0$, which of the following is **true?**

(a) $3^x > 4^x$ **(b)** $7^x > 5^x$ **(c)** $(1/6)^x > (1/2)^x$
(d) $9^{-x} > 8^{-x}$ **(e)** $0.17^x > 0.32^x$

Explorations

65. Graph each function and analyze it for domain, range, increasing or decreasing behavior, boundedness, extrema, asymptotes, and end behavior.

(a) $f(x) = x \cdot e^x$ **(b)** $g(x) = \dfrac{e^{-x}}{x}$

66. Use the properties of exponents to solve each equation. Support graphically.

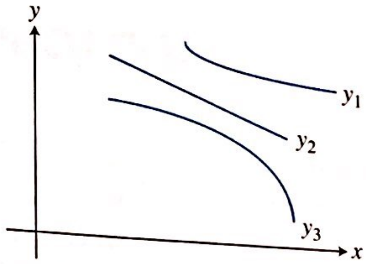
(a) $2^x = 4^2$ **(b)** $3^x = 27$
(c) $8^{x/2} = 4^{x+1}$ **(d)** $9^x = 3^{x+1}$

Extending the Ideas

67. Writing to Learn Table 3.8 gives function values for $y = f(x)$ and $y = g(x)$. Also, three different graphs are shown.

Table 3.8 Data for Two Functions

x	$f(x)$	$g(x)$
1.0	5.50	7.40
1.5	5.35	6.97
2.0	5.25	6.44
2.5	5.17	5.76
3.0	5.13	4.90
3.5	5.09	3.82
4.0	5.06	2.44
4.5	5.05	0.71



(a) Which curve of those shown in the graph most closely resembles the graph of $y = f(x)$? Explain your choice.

(b) Which curve most closely resembles the graph of $y = g(x)$? Explain your choice.

68. Writing to Learn Let $f(x) = 2^x$. Explain why the graph of $f(ax + b)$ can be obtained by applying one transformation to the graph of $y = c^x$ for an appropriate value of c. What is c?

Exercises 69–72 refer to the expression $f(a, b, c) = a \cdot b^c$. If $a = 2$, $b = 3$, and $c = x$, the expression is $f(2, 3, x) = 2 \cdot 3^x$, an exponential function.

69. If $b = x$, state conditions on a and c under which the expression $f(a, b, c)$ is a quadratic power function.

70. If $b = x$, state conditions on a and c under which the expression $f(a, b, c)$ is a decreasing linear function.

71. If $c = x$, state conditions on a and b under which the expression $f(a, b, c)$ is an increasing exponential function.

72. If $c = x$, state conditions on a and b under which the expression $f(a, b, c)$ is a decreasing exponential function.

73. Prove that $\lim_{x\to-\infty} \dfrac{c}{1 + a \cdot b^x} = 0$ and $\lim_{x\to\infty} \dfrac{c}{1 + a \cdot b^x} = c$, for constants $a, b, c > 0$ with $b < 1$.