SECTION 4.3 EXERCISES

In Exercises 1 and 2, identify the one angle that is not coterminal with all the others.

1. $150°, 510°, -210°, 450°, 870°$

2. $\frac{5\pi}{3}, -\frac{5\pi}{3}, \frac{11\pi}{3}, -\frac{7\pi}{3}, \frac{365\pi}{3}$

In Exercises 3–6, evaluate the six trigonometric functions of the angle θ.

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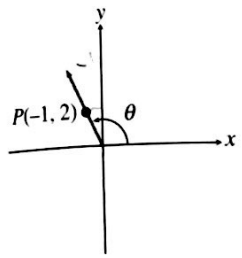

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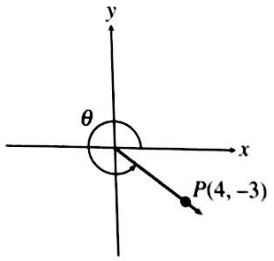

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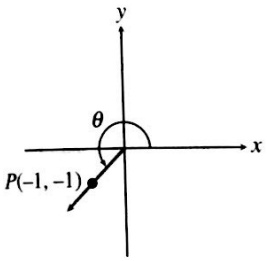

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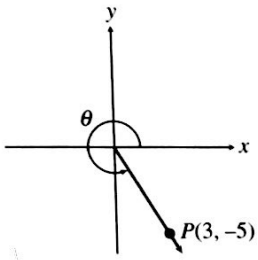


In Exercises 7–12, point P is on the terminal side of angle θ. Evaluate the six trigonometric functions for θ. If the function is undefined, write "undefined."

7. $P(3, 4)$
8. $P(-4, -6)$
9. $P(0, 5)$
10. $P(-3, 0)$
11. $P(5, -2)$
12. $P(22, -22)$

In Exercises 13–16, state the sign (+ or −) of **(a)** $\sin t$, **(b)** $\cos t$, and **(c)** $\tan t$ for values of t in the interval given.

13. $\left(0, \frac{\pi}{2}\right)$
14. $\left(\frac{\pi}{2}, \pi\right)$
15. $\left(\pi, \frac{3\pi}{2}\right)$
16. $\left(\frac{3\pi}{2}, 2\pi\right)$

In Exercises 17–20, determine the sign (+ or −) of the given value without the use of a calculator.

17. $\cos 143°$
18. $\tan 192°$
19. $\cos \frac{7\pi}{8}$
20. $\tan \frac{4\pi}{5}$

In Exercises 21–24, choose the point on the terminal side of θ.

21. $\theta = 45°$

(a) $(2, 2)$ **(b)** $(1, \sqrt{3})$ **(c)** $(\sqrt{3}, 1)$

22. $\theta = \frac{2\pi}{3}$

(a) $(-1, 1)$ **(b)** $(-1, \sqrt{3})$ **(c)** $(-\sqrt{3}, 1)$

23. $\theta = \frac{7\pi}{6}$

(a) $(-\sqrt{3}, -1)$ **(b)** $(-1, \sqrt{3})$ **(c)** $(-\sqrt{3}, 1)$

24. $\theta = -60°$

(a) $(-1, -1)$ **(b)** $(1, -\sqrt{3})$ **(c)** $(-\sqrt{3}, 1)$

In Exercises 25–36, evaluate without using a calculator by using ratios in a reference triangle.

25. $\cos 120°$
26. $\tan 300°$
27. $\sec \frac{\pi}{3}$
28. $\csc \frac{3\pi}{4}$
29. $\sin \frac{13\pi}{6}$
30. $\cos \frac{7\pi}{3}$
31. $\tan -\frac{15\pi}{4}$
32. $\cot \frac{13\pi}{4}$
33. $\cos \frac{23\pi}{6}$
34. $\cos \frac{17\pi}{4}$
35. $\sin \frac{11\pi}{3}$
36. $\cot \frac{19\pi}{6}$

In Exercises 37–42, find **(a)** $\sin\theta$, **(b)** $\cos\theta$, and **(c)** $\tan\theta$ for the given quadrantal angle. If the value is undefined, write "undefined."

37. $-450°$
38. $-270°$
39. 7π
40. $\frac{11\pi}{2}$
41. $-\frac{7\pi}{2}$
42. -4π

In Exercises 43–48, evaluate without using a calculator.

43. Find $\sin\theta$ and $\tan\theta$ if $\cos\theta = \frac{2}{3}$ and $\cot\theta > 0$.

44. Find $\cos\theta$ and $\cot\theta$ if $\sin\theta = \frac{1}{4}$ and $\tan\theta < 0$.

45. Find $\tan\theta$ and $\sec\theta$ if $\sin\theta = -\frac{2}{5}$ and $\cos\theta > 0$.

46. Find $\sin\theta$ and $\cos\theta$ if $\cot\theta = \frac{3}{7}$ and $\sec\theta < 0$.

47. Find $\sec\theta$ and $\csc\theta$ if $\cot\theta = -\frac{4}{3}$ and $\cos\theta < 0$.

48. Find $\csc\theta$ and $\cot\theta$ if $\tan\theta = -\frac{4}{3}$ and $\sin\theta > 0$.

In Exercises 49–52, evaluate by using the period of the function.

49. $\sin\left(\frac{\pi}{6} + 49{,}000\pi\right)$

50. $\tan(1{,}234{,}567\pi) - \tan(7{,}654{,}321\pi)$

51. $\cos\left(\frac{5{,}555{,}555\pi}{2}\right)$

52. $\tan\left(\frac{3\pi - 70{,}000\pi}{2}\right)$

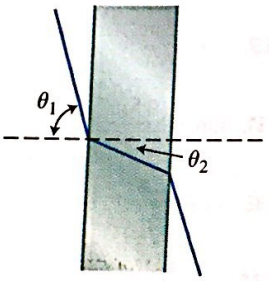
53. Group Activity Use a calculator to evaluate the expressions in Exercises 49–52. Does your calculator give the correct answers? Many calculators miss all four. Give a brief explanation of what probably goes wrong.

54. Writing to Learn Give a convincing argument that the period of $\sin t$ is 2π. That is, show that there is no smaller positive real number p such that $\sin(t + p) = \sin t$ for all real numbers t.

55. Refracted Light Light is *refracted* (bent) as it passes through glass. In the figure below θ_1 is the angle of incidence and θ_2 is the *angle of refraction*. The *index of refraction* is a constant μ that satisfies the equation

$$\sin\theta_1 = \mu \sin\theta_2.$$

If $\theta_1 = 83°$ and $\theta_2 = 36°$ for a certain piece of flint glass, find the index of refraction.



56. Refracted Light A certain piece of crown glass has an index of refraction of 1.52. If a light ray enters the glass at an angle $\theta_1 = 42°$, what is $\sin\theta_2$?

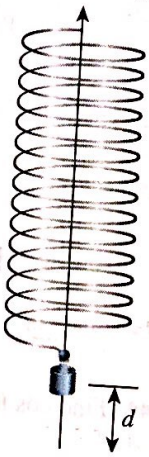
57. Damped Harmonic Motion A weight suspended from a spring is set into motion. Its dispacement d from equilibrium is modeled by the equation

$$d = 0.4e^{-0.2t}\cos 4t.$$

where d is the displacement in inches and t is the time in seconds. Find the displacement at the given time. Use radian mode.

(a) $t = 0$

(b) $t = 3$



58. Swinging Pendulum The Columbus Museum of Science and Industry exhibits a Foucault pendulum 32 ft long that swings back and forth on a cable once in approximately 6 sec. The angle θ (in radians) between the cable and an imaginary vertical line is modeled by the equation

$$\theta = 0.25\cos t.$$

Find the measure of angle θ when $t = 0$ and $t = 2.5$.

59. Too Close for Comfort An F-15 aircraft flying at an altitude of 8000 ft passes directly over a group of vacationers hiking at 7400 ft. If θ is the angle of elevation from the hikers to the F-15, find the distance d from the group to the jet for the given angle.

(a) $\theta = 45°$ **(b)** $\theta = 90°$ **(c)** $\theta = 140°$

60. Manufacturing Swimwear Get Wet, Inc. manufactures swimwear, a seasonal product. The monthly sales x (in thousands) for Get Wet swimsuits are modeled by the equation

$$x = 72.4 + 61.7\sin\frac{\pi t}{6},$$

where $t = 1$ represents January, $t = 2$ February, and so on. Estimate the number of Get Wet swimsuits sold in January, April, June, October, and December. For which two of these months are sales the same? Explain why this might be so.

Standardized Test Questions

61. True or False If θ is an angle of a triangle such that $\cos\theta < 0$, then θ is obtuse. Justify your answer.

62. True or False If θ is an angle in standard position determined by the point $(8,-6)$, then $\sin\theta = -0.6$. Justify your answer.

You should answer these questions without using a calculator.

63. Multiple Choice If $\sin\theta = 0.4$, then $\sin(-\theta) + \csc\theta =$

(a) -0.15 **(b)** 0 **(c)** 0.15 **(d)** 0.65 **(e)** 2.1

64. Multiple Choice If $\cos\theta = 0.4$, then $\cos(\theta + \pi) =$

(a) -0.6 **(b)** -0.4 **(c)** 0.4 **(d)** 0.6 **(e)** 3.54

65. Multiple Choice The range of the function $f(t) = (\sin t)^2 + (\cos t)^2$ is

(a) $\{1\}$ **(b)** $[-1, 1]$ **(c)** $[0, 1]$

(d) $[0, 2]$ **(e)** $[0, \infty)$

66. Multiple Choice If $\cos\theta = -\frac{5}{13}$ and $\tan\theta > 0$, then $\sin\theta =$

(a) $-\frac{12}{13}$ **(b)** $-\frac{5}{12}$ **(c)** $\frac{5}{13}$ **(d)** $\frac{5}{12}$ **(e)** $\frac{12}{13}$

Explorations

In Exercises 67–70, find the value of the unique real number θ between 0 and 2π that satisfies the two given conditions.

67. $\sin\theta = \frac{1}{2}$ and $\tan\theta < 0$.

68. $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta < 0$.

69. $\tan\theta = -1$ and $\sin\theta < 0$.

70. $\sin\theta = -\frac{\sqrt{2}}{2}$ and $\tan\theta > 0$.