CHAPTER 4 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–8, determine the quadrant of the terminal side of the angle in standard position. Convert degree measures to radians and radian measures to degrees.

1. $\frac{5\pi}{2}$
2. $\frac{3\pi}{4}$
3. $-135°$
4. $-45°$
5. $78°$
6. $112°$
7. $\frac{\pi}{12}$
8. $\frac{7\pi}{10}$

In Exercises 9 and 10, determine the angle measure in both degrees and radians. Draw the angle in standard position if its terminal side is obtained as described.

9. A three-quarters counterclockwise rotation
10. Two and one-half counterclockwise rotations

In Exercises 11–16, the point is on the terminal side of an angle in standard position. Give the smallest positive angle measure in both degrees and radians.

11. $(\sqrt{3}, 1)$
12. $(-1, 1)$
13. $(-1, \sqrt{3})$
14. $(-3, -3)$
15. $(6, -12)$
16. $(2, 4)$

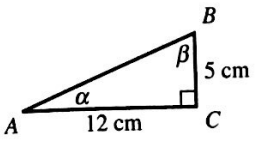
In Exercises 17–28, evaluate the expression exactly without a calculator.

17. $\sin 30°$
18. $\cos 330°$
19. $\tan(-135°)$
20. $\sec(-135°)$
21. $\sin\frac{5\pi}{6}$
22. $\csc\frac{2\pi}{3}$
23. $\sec(-\frac{\pi}{3})$
24. $\tan(-\frac{2\pi}{3})$
25. $\csc 270°$
26. $\sec 180°$
27. $\cot(-90°)$
28. $\tan 360°$

In Exercises 29–32, evaluate exactly all six trigonometric functions of the angle. Use reference triangles and not your calculator.

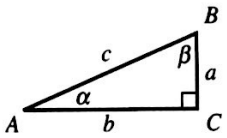
29. $-\frac{\pi}{6}$
30. $\frac{19\pi}{4}$
31. $-135°$
32. $420°$

33. Find all six trigonometric functions of α in $\triangle ABC$.



34. Use a right triangle to determine the values of all trigonometric functions of θ, where $\cos\theta = 5/7$.
35. Use a right triangle to determine the values of all trigonometric functions of θ, where $\tan\theta = 15/8$.
36. Use a calculator in degree mode to solve $\cos\theta = 3/7$ if $0° \le \theta \le 90°$.
37. Use a calculator in radian mode to solve $\tan x = 1.35$ if $\pi \le x \le 3\pi/2$.
38. Use a calculator in radian mode to solve $\sin x = 0.218$ if $0 \le x \le 2\pi$.

In Exercises 39–44, solve the right $\triangle ABC$.



39. $\alpha = 35°,\ c = 15$
40. $b = 8,\ c = 10$
41. $\beta = 48°,\ a = 7$
42. $\alpha = 28°,\ c = 8$
43. $b = 5,\ c = 7$
44. $a = 2.5,\ b = 7.3$

In Exercises 45–48, x is an angle in standard position with $0 \le x \le 2\pi$. Determine the quadrant of x.

45. $\sin x < 0$ and $\tan x > 0$
46. $\cos x < 0$ and $\csc x > 0$
47. $\tan x < 0$ and $\sin x > 0$
48. $\sec x < 0$ and $\csc x > 0$

In Exercises 49–52, point P is on the terminal side of angle θ. Evaluate the six trigonometric functions for θ.

49. $(-3, 6)$
50. $(12, 7)$
51. $(-5, -3)$
52. $(4, 9)$

In Exercises 53–60, use transformations to describe how the graph of the function is related to a basic trigonometric graph. Graph two periods.

53. $y = \sin(x + \pi)$
54. $y = 3 + 2\cos x$
55. $y = -\cos(x + \pi/2) + 4$
56. $y = -2 - 3\sin(x - \pi)$
57. $y = \tan 2x$
58. $y = -2\cot 3x$
59. $y = -2\sec\frac{x}{2}$
60. $y = \csc \pi x$

In Exercises 61–66, state the amplitude, period, phase shift, domain, and range for the sinusoid.

61. $f(x) = 2\sin 3x$
62. $g(x) = 3\cos 4x$
63. $f(x) = 1.5\sin(2x - \pi/4)$
64. $g(x) = -2\sin(3x - \pi/3)$
65. $y = 4\cos(2x - 1)$
66. $g(x) = -2\cos(3x + 1)$

In Exercises 67 and 68, graph the function. Then estimate the values of a, b, and h so that $f(x) \approx a\sin(b(x - h))$.

67. $f(x) = 2\sin x - 4\cos x$
68. $f(x) = 3\cos 2x - 2\sin 2x$

In Exercises 69–72, use a calculator to evaluate the expression. Express your answer in both degrees and radians.

69. $\sin^{-1}(0.766)$

70. $\cos^{-1}(0.479)$

71. $\tan^{-1}1$

72. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

In Exercises 73–76, use transformations to describe how the graph of the function is related to a basic inverse trigonometric graph. State the domain and range.

73. $y = \sin^{-1} 3x$

74. $y = \tan^{-1} 2x$

75. $y = \sin^{-1}(3x - 1) + 2$

76. $y = \cos^{-1}(2x + 1) - 3$

In Exercises 77–82, find the exact value of x without using a calculator.

77. $\sin x = 0.5, \quad \pi/2 \le x \le \pi$

78. $\cos x = \sqrt{3}/2, \quad 0 \le x \le \pi$

79. $\tan x = -1, \quad 0 \le x \le \pi$

80. $\sec x = 2, \quad \pi \le x \le 2\pi$

81. $\csc x = -1, \quad 0 \le x \le 2\pi$

82. $\cot x = -\sqrt{3}, \quad 0 \le x \le \pi$

In Exercises 83 and 84, describe the end behavior of the function.

83. $\frac{\sin x}{x^2}$

84. $\frac{3}{5}e^{-x/12}\sin(2x - 3)$

In Exercises 85–88, evaluate the expression without a calculator.

85. $\tan(\tan^{-1} 1)$

86. $\cos^{-1}(\cos \pi/3)$

87. $\tan(\sin^{-1} 3/5)$

88. $\cos^{-1}(\cos -\pi/7)$

In Exercises 89–92, determine whether the function is periodic. State the period (if applicable), the domain, and the range.

89. $f(x) = |\sec x|$

90. $g(x) = \sin|x|$

91. $f(x) = 2x + \tan x$

92. $g(x) = 2\cos 2x + 3\sin 5x$

93. Arc Length Find the length of the arc intercepted by a central angle of $2\pi/3$ rad in a circle with radius 2.

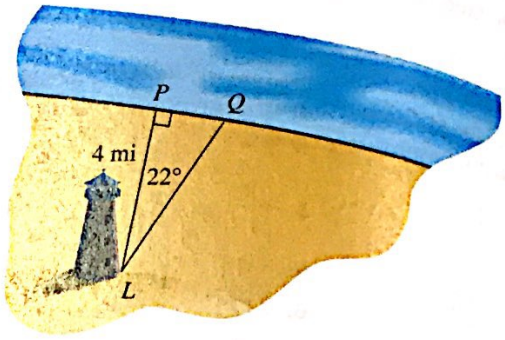
94. Algebraic Expression Find an algebraic expression equivalent to $\tan(\cos^{-1} x)$.

95. Height of Building The angle of elevation of the top of a building from a point 100 m away from the building on level ground is 78°. Find the height of the building.

96. Height of Tree A tree casts a shadow 51 ft long when the angle of elevation of the sun (measured with the horizon) is 25°. How tall is the tree?

97. Traveling Car From the top of a 150-ft building Flora observes a car moving toward her. If the angle of depression of the car changes from 18° to 42° during the observation, how far does the car travel?

98. Finding Distance A lighthouse L stands 4 mi from the closest point P along a straight shore (see figure). Find the distance from P to a point Q along the shore if $\angle PLQ = 22°$.



99. Navigation An airplane is flying due east between two signal towers. One tower is due north of the other. The bearing from the plane to the north tower is 23°, and to the south tower is 128°. Use a drawing to show the exact location of the plane.

100. Finding Distance The bearings of two points on the shore from a boat are 115° and 123°. Assume the two points are 855 ft apart. How far is the boat from the nearest point on shore if the shore is straight and runs north-south?

101. Height of Tree Dr. Thom Lawson standing on flat ground 62 ft from the base of a Douglas fir measures the angle of elevation to the top of the tree as 72°24′. What is the height of the tree?

102. Storing Hay A 75-ft-long conveyor is used at the Lovelady Farm to put hay bales up for winter storage. The conveyor is tilted to an angle of elevation of 22°.

(a) To what height can the hay be moved?

(b) If the conveyor is repositioned to an angle of 27°, to what height can the hay be moved?

103. Swinging Pendulum In the Hardy Boys Adventure *While the Clock Ticked*, the pendulum of the grandfather clock at the Purdy place is 44 in. long and swings through an arc of 6°. Find the length of the arc that the pendulum traces.

104. Finding Area A windshield wiper on a Plymouth Acclaim is 20 in. long and has a blade 16 in. long. If the wiper sweeps through an angle of 110°, how large an area does the wiper blade clean? (See Exercise 71 in Section 4.1.)

105. Modeling Mean Temperature The average daily air temperature (°F) for Fairbanks, Alaska, from 1941 to 1970, can be modeled by the equation

$$T(x) = 37\sin\left[\frac{2\pi}{365}(x - 101)\right] + 25,$$

where x is time in days with $x = 1$ representing January 1. On what days do you expect the average temperature to be 32°F?

106. **Taming The Beast** The Beast is a featured roller coaster at the King Island's amusement park just north of Cincinnati. On its first and biggest hill, The Beast drops from a height of 52 ft above the ground along a sinusoidal path to a depth 18 ft underground as it enters a frightening tunnel. The mathematical model for this part of track is

$$h(x) = 35 \cos\left(\frac{x}{35}\right) + 17, 0 \leq x \leq 110,$$

where x is the horizontal distance from the top of the hill and $h(x)$ is the vertical position relative to ground level (both in feet). What is the horizontal distance from the top of the hill to the point where the track reaches ground level?