

SECTION 4.4 EXERCISES

In Exercises 1–6, find the amplitude of the function and use the language of transformations to describe how the graph of the function is related to the graph of $y = \sin x$.

1. $y = 2 \sin x$
2. $y = \frac{2}{3} \sin x$
3. $y = -4 \sin x$
4. $y = -\frac{7}{4} \sin x$
5. $y = 0.73 \sin x$
6. $y = -2.34 \sin x$

In Exercises 7–12, find the period of the function and use the language of transformations to describe how the graph of the function is related to the graph of $y = \cos x$.

7. $y = \cos 3x$
8. $y = \cos x/5$
9. $y = \cos(-7x)$
10. $y = \cos(-0.4x)$
11. $y = 3 \cos 2x$
12. $y = \frac{1}{4} \cos \frac{2x}{3}$

In Exercises 13–16, find the amplitude, period, and frequency of the function and use this information (not your calculator) to sketch a graph of the function in the window $[-3\pi, 3\pi]$ by $[-4, 4]$.

13. $y = 3 \sin \frac{x}{2}$
14. $y = 2 \cos \frac{x}{3}$
15. $y = -\frac{3}{2} \sin 2x$
16. $y = -4 \sin \frac{2x}{3}$

In Exercises 17–22, graph one period of the function. Use your understanding of transformations, not your graphing calculators. Be sure to show the scale on both axes.

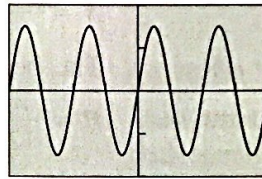
17. $y = 2 \sin x$
18. $y = 2.5 \sin x$
19. $y = 3 \cos x$
20. $y = -2 \cos x$
21. $y = -0.5 \sin x$
22. $y = 4 \cos x$

In Exercises 23–28, graph three periods of the function. Use your understanding of transformations, not your graphing calculators. Be sure to show the scale on both axes.

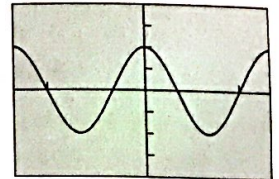
23. $y = 5 \sin 2x$
24. $y = 3 \cos \frac{x}{2}$
25. $y = 0.5 \cos 3x$
26. $y = 20 \sin 4x$
27. $y = 4 \sin \frac{x}{4}$
28. $y = 8 \cos 5x$

In Exercises 29–34, specify the period and amplitude of each function. Then give the viewing window in which the graph is shown. Use your understanding of transformations, not your graphing calculators.

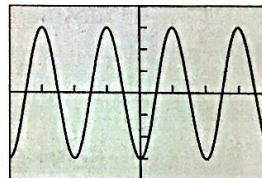
29. $y = 1.5 \sin 2x$



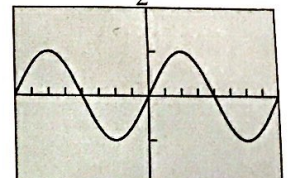
30. $y = 2 \cos 3x$



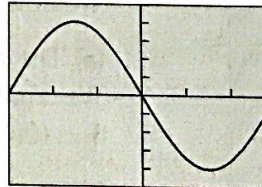
31. $y = -3 \cos 2x$



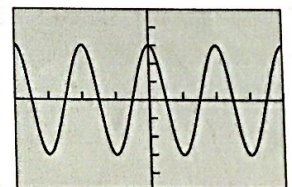
32. $y = 5 \sin \frac{x}{2}$



33. $y = -4 \sin \frac{\pi}{3}x$



34. $y = 3 \cos \pi x$



In Exercises 35–40, identify the maximum and minimum values and the zeros of the function in the interval $[-2\pi, 2\pi]$. Use your understanding of transformations, not your graphing calculators.

35. $y = 2 \sin x$
36. $y = 3 \cos \frac{x}{2}$
37. $y = \cos 2x$
38. $y = \frac{1}{2} \sin x$
39. $y = -\cos 2x$
40. $y = -2 \sin x$

41. Write the function $y = -\sin x$ as a phase shift of $y = \sin x$.
42. Write the function $y = -\cos x$ as a phase shift of $y = \sin x$.

In Exercises 43–48, describe the transformations required to obtain the graph of the given function from a basic trigonometric graph.

43. $y = 0.5 \sin 3x$

44. $y = 1.5 \cos 4x$

45. $y = -\frac{2}{3} \cos \frac{x}{3}$

46. $y = \frac{3}{4} \sin \frac{x}{5}$

47. $y = 3 \cos \frac{2\pi x}{3}$

48. $y = -2 \sin \frac{\pi x}{4}$

In Exercises 49–52, describe the transformations required to obtain the graph of y_2 from the graph of y_1 .

49. $y_1 = \cos 2x$ and $y_2 = \frac{5}{3} \cos 2x$

50. $y_1 = 2 \cos \left(x + \frac{\pi}{3}\right)$ and $y_2 = \cos \left(x + \frac{\pi}{4}\right)$

51. $y_1 = 2 \cos \pi x$ and $y_2 = 2 \cos 2\pi x$

52. $y_1 = 3 \sin \frac{2\pi x}{3}$ and $y_2 = 2 \sin \frac{\pi x}{3}$

In Exercises 53–56, select the pair of functions that have identical graphs.

53. (a) $y = \cos x$ (b) $y = \sin \left(x + \frac{\pi}{2}\right)$

(c) $y = \cos \left(x + \frac{\pi}{2}\right)$

54. (a) $y = \sin x$

(b) $y = \cos \left(x - \frac{\pi}{2}\right)$

(c) $y = \cos x$

55. (a) $y = \sin \left(x + \frac{\pi}{2}\right)$

(b) $y = -\cos(x - \pi)$

(c) $y = \cos \left(x - \frac{\pi}{2}\right)$

56. (a) $y = \sin \left(2x + \frac{\pi}{4}\right)$

(b) $y = \cos \left(2x - \frac{\pi}{2}\right)$

(c) $y = \cos \left(2x - \frac{\pi}{4}\right)$

In Exercises 57–60, construct a sinusoid with the given amplitude and period that goes through the given point.

57. Amplitude 3, period π , point $(0, 0)$

58. Amplitude 2, period 3π , point $(0, 0)$

59. Amplitude 1.5, period $\pi/6$, point $(1, 0)$

60. Amplitude 3.2, period $\pi/7$, point $(5, 0)$

In Exercises 61–68, state the amplitude and period of the sinusoid, and (relative to the basic function) the phase shift and vertical translation.

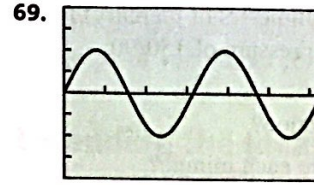
61. $y = -2 \sin \left(x - \frac{\pi}{4}\right) + 1$ 62. $y = -3.5 \sin \left(2x - \frac{\pi}{2}\right) - 1$

63. $y = 5 \cos \left(3x - \frac{\pi}{6}\right) + 0.5$ 64. $y = 3 \cos(x + 3) - 2$

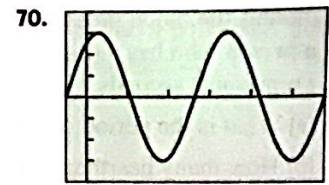
65. $y = 2 \cos 2\pi x + 1$ 66. $y = 4 \cos 3\pi x - 2$

67. $y = \frac{7}{3} \sin \left(x + \frac{5}{2}\right) - 1$ 68. $y = \frac{2}{3} \cos \left(\frac{x-3}{4}\right) + 1$

In Exercises 69 and 70, find values a , b , h , and k so that the graph of the function $y = a \sin(b(x+h)) + k$ is the curve shown.



[0, 6.28] by [-4, 4]



[-0.5, 5.78] by [-4, 4]

71. **Points of Intersection** Graph $y = 1.3^{-x}$ and $y = 1.3^{-x} \cos x$ for x in the interval $[-1, 8]$.

(a) How many points of intersection do there appear to be?

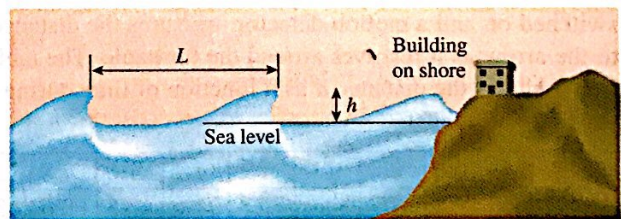
(b) Find the coordinates of each point of intersection.

72. **Motion of a Buoy** A signal buoy in the Chesapeake Bay bobs up and down with the height h of its transmitter (in feet) above sea level modeled by $h = a \sin bt + 5$. During a small squall its height varies from 1 ft to 9 ft and there are 3.5 sec from one 9-ft height to the next. What are the values of the constants a and b ?



73. **Ferris Wheel** A Ferris wheel 50 ft in diameter makes one revolution every 40 sec. If the center of the wheel is 30 ft above the ground, how long after reaching the low point is a rider 50 ft above the ground?

74. **Tsunami Wave** An earthquake occurred at 9:40 A.M. on Nov. 1, 1755, at Lisbon, Portugal, and started a *tsunami* (often called a tidal wave) in the ocean. It produced waves that traveled more than 540 ft/sec (370 mph) and reached a height of 60 ft. If the period of the waves was 30 min or 1800 sec, estimate the length L between the crests.



75. **Ebb and Flow** On a particular Labor Day, the high tide in southern California occurs at 7:12 A.M. At that time you measure the water at the end of the Santa Monica Pier to be 11 ft deep. At 1:24 P.M. it is low tide, and you measure the water to be only 7 ft deep. Assume the depth of the water is a sinusoidal function of time with a period of $1/2$ a lunar day, which is about 12 h 24 min.

(a) At what time on that Labor Day does the first low tide occur?

(b) What was the approximate depth of the water at 4:00 A.M. and at 9:00 P.M.?

(c) What is the first time on that Labor Day that the water is 9 ft deep?