

Pre-Calculus Chapter 4 Review

1. Evaluate the following:

a. $\sin 30^\circ$ val.

$\frac{1}{2}$

b. $\tan\left(-\frac{2\pi}{3}\right)$ $\frac{y}{x}$

$-\sqrt{3}$

c. $\csc \frac{5\pi}{6}$

$\frac{1}{\sin\left(\frac{5\pi}{6}\right)}$

2

$\frac{1}{\frac{1}{2}} = 2$

2. Evaluate all six trig functions of the angle $-\frac{\pi}{6}$.

$\sin \theta = \frac{-1/2}{\sqrt{3/2}}$

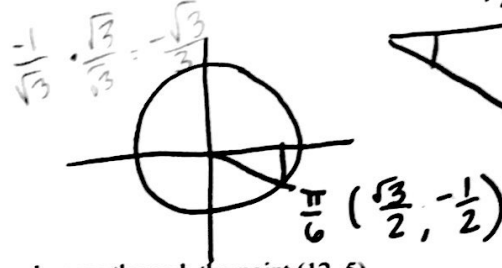
$\csc \theta = \frac{-2}{\sqrt{3/2}}$

$\cos \theta = \frac{\sqrt{3/2}}{\sqrt{3/2}}$

$\sec \theta = \frac{2\sqrt{3/2}}{\sqrt{3/2}}$

$\tan \theta = \frac{-1/2}{\sqrt{3/2}}$

$\cot \theta = \frac{-\sqrt{3}}{1}$



3. Find all six trig functions when the terminal side of the angle goes through the point (12, 5).

$\sin \theta = \frac{5}{13}$

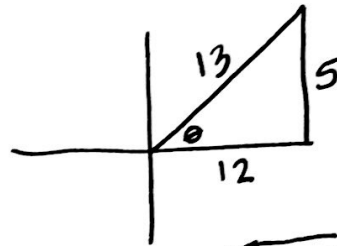
$\csc \theta = \frac{13}{5}$

$\cos \theta = \frac{12}{13}$

$\sec \theta = \frac{13}{12}$

$\tan \theta = \frac{5}{12}$

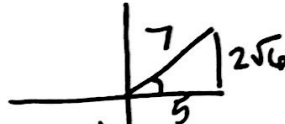
$\cot \theta = \frac{12}{5}$



4. If $\cos \theta = \frac{5}{7}$ and $\tan \theta > 0$, find the values of sine and tangent.

$\sin \theta = \frac{2\sqrt{6}}{7}$

$\tan \theta = \frac{2\sqrt{6}}{5}$



$\sqrt{7^2 - 5^2} = \sqrt{24} = 2\sqrt{6}$

5. Given $f(x) = 8 \cos(2x + \pi) - 3 \rightarrow 8 \cos\left(2\left(x + \frac{\pi}{2}\right)\right) - 3$

a. Amplitude: 8

b. Period: $\frac{2\pi}{2} = \pi$

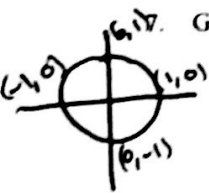
c. Phase Shift: $\frac{\pi}{2}$ left

d. Vertical Shift: Down 3

6. Given $f(x) = 4 \cos\left(3x - \frac{\pi}{3}\right) + 1$

a. Domain: $(-\infty, \infty)$

b. Range: $[-3, 5]$



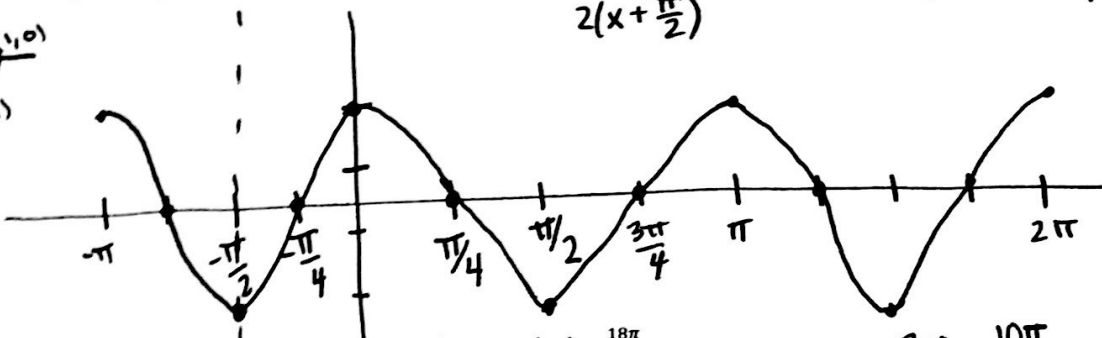
7. Graph three periods of the function: $f(x) = -2 \cos(2x + \pi)$

$2\left(x + \frac{\pi}{2}\right)$

A: 2

P: π

PS: $\frac{\pi}{2}$ L



$-\pi$	
$-\frac{3\pi}{2}$	
$-\pi/2$	$-2(1) = -2$
0	$-2(0) = 0$
$\pi/4$	$-2(-1) = 2$
$\pi/2$	$-2(0) = 0$
$3\pi/4$	$-2(1) = -2$
π	
2π	

8. Find one positive and one negative angle coterminal to $\frac{18\pi}{5}$

$\frac{18\pi}{5} - \frac{10\pi}{5} = \frac{8\pi}{5}$ $-\frac{10\pi}{5} = -\frac{2\pi}{5}$ $\frac{2\pi}{1} = \frac{10\pi}{5}$

a. Positive: $\frac{8\pi}{5}$

b. Negative: $-\frac{2\pi}{5}$

Pre-Calculus Chapter 4 Review

9. Evaluate the following (b and c should be done without a calculator):

a. $\sin^{-1}(0.766)$

b. $\tan^{-1} 1$

c. $\sin^{-1} \frac{\sqrt{3}}{2}$

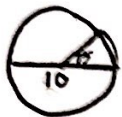
50° or $.87$ rad.

$\frac{\pi}{4}$ or 45°

$\frac{\pi}{3}$ or 60°

10. The other day, my friend Erica bought a block of fancy cheese. The wedge had an angle of 15° and came from a wheel with a diameter of 10 inches. Find the perimeter of the wedge of cheese.

$2\pi r \cdot \frac{15}{360}$



$\theta = 15^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{12}$

$P = 5 + 5 + 1.31$

$P = 11.31$

Perimeter = 11.31

$S = r\theta = 5\left(\frac{\pi}{12}\right) = 1.31$

Arc length = $r\theta$

11. Solve for x on the interval $[0, \frac{\pi}{2}]$.

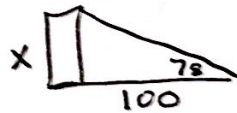
a. $\sec x = 2$

b. $\sin x = .5$

$\frac{\pi}{3}$

$\frac{\pi}{6}$

12. The angle of elevation to Mrs. Leaptrott's classroom from a point 100 feet away is 78° . Determine how high it is from the base of the building to Mrs. Leaptrott's room.



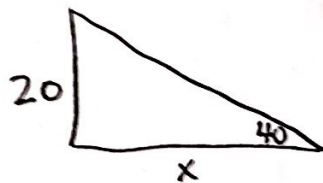
$\tan 78 = \frac{x}{100}$

$100 \tan(78) = x$

$470.46 = x$

Height = 470.46 ft

13. You are trying to throw popcorn into your friend's mouth from a balcony 20 feet high. The angle of depression from you to your friend is 40° . How far away does your friend need to be to catch it?



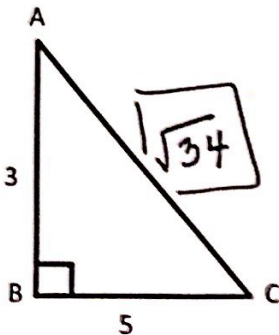
$\tan 40 = \frac{20}{x}$

$x \tan 40 = 20$

$x = \frac{20}{\tan 40} = 23.84$

Distance = 23.84 ft

14. Solve the triangle for all angles and sides:



$3^2 + 5^2 = c^2$

$\sqrt{34} = \sqrt{c^2}$

$\angle A: \tan A = \frac{5}{3}$

$\angle C: \tan \frac{3}{5}$

$\tan^{-1}(\frac{5}{3}) =$

$\tan^{-1}(\frac{3}{5}) =$

59.04°

30.96°

15. Solve (Hint: use your calculator...graphing might help)

a. $\sec x = 5$ $0 < x < \pi$

b. $\tan x = \sin x$ $-\pi < x \leq \pi$

$y_1 = \tan x$

$y_2 = \sin x$

$y_1 = \frac{1}{\cos(x)}$
 $y_2 = 5$

Zoom trig 1.37 rad

$[0, \pi]$

Pre-Calculus: Chapter 4 Trig Review Sheet

Calculator INACTIVE

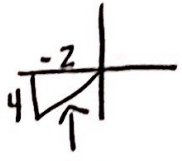
1. Consider an angle whose terminal side passes through the point (-2, -4). Find the following:

a. $\tan \theta = 2$

b. $\csc \theta = \frac{2\sqrt{5}}{-4} = \boxed{-\frac{\sqrt{5}}{2}}$

c. $\cot \theta = \frac{1}{2}$

d. $\frac{\sin \theta}{2\sqrt{5}} = \frac{-4}{2\sqrt{5}} = \boxed{-\frac{2\sqrt{5}}{5}}$



2. Consider the angle $\theta = \frac{42\pi}{5}$.

Find one positive and one negative coterminal angle in radian form in the interval $[-2\pi, 2\pi]$.

$\frac{42\pi}{5} - \frac{10\pi}{5} = \frac{32\pi}{5} - \frac{10\pi}{5} = \frac{22\pi}{5} - \frac{10\pi}{5} = \frac{12\pi}{5} - \frac{10\pi}{5} = \frac{2\pi}{5}$

$-\frac{10\pi}{5} = \boxed{-\frac{8\pi}{5}}$

3. Find the exact value of the following unit circle points:

a. $\tan \frac{\pi}{2} = \frac{1}{0}$ und.

b. $\sin \frac{-2\pi}{3} = -\frac{\sqrt{3}}{2}$

c. $\sec \frac{5\pi}{4} = -\sqrt{2}$

d. $\csc \frac{3\pi}{2} = \frac{1}{-1} = \boxed{-1}$

e. $\cot \pi = \frac{0}{0} = \text{undefined}$

f. $\cos \frac{-\pi}{4} = \frac{\sqrt{2}}{2}$

g. $\sin 420^\circ = \frac{\sqrt{3}}{2}$

4. State the domain and range in interval notation for each trigonometric function:

a. $\cos x$

b. $\sin^{-1} x$

c. $\sec x$

$D: (-\infty, -1] \cup [1, \infty)$

$D: (-\infty, \infty)$ $R: [-1, 1]$

$D: [-1, 1]$ $R: [-\frac{\pi}{2}, \frac{\pi}{2}]$

$D: (-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2})$

5. If $\sin \theta > 0$ and $\tan \theta = -\frac{6}{2}$, find the exact values of the following:

a. $\sin \theta = \frac{6}{2\sqrt{10}} = \frac{3\sqrt{10}}{10}$

b. $\cot \theta = \frac{-2}{6} = -\frac{1}{3}$

c. $\sec \theta = \frac{2\sqrt{10}}{-2} = -\sqrt{10}$



6. State the exact values of the following:

a. $\arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}$

b. $\arccos -\frac{1}{2} = \frac{2\pi}{3}$

c. $\tan^{-1} 1 = \frac{\pi}{4}$

d. $\arctan(-\frac{1}{\sqrt{3}}) = -\frac{\pi}{6}$

7. State the period, phase shift, amplitude, and vertical shift of $f(x) = 3 \cos(2x + \frac{\pi}{2}) + 1$. Graph the function.

on other sheet

8. Write the equation of a sine function that has an amplitude of 4, period of $\frac{3\pi}{2}$, a phase shift π units to the right, and a vertical shift down 7 units.

$y = 4 \sin(\frac{4}{3}(x - \pi)) - 7$

$\frac{2\pi}{b} = \frac{3\pi}{2} \rightarrow 4\pi = 3\pi b$
 $b = \frac{4}{3}$

9. Where on the interval $[-2\pi, 2\pi]$ are the following trig functions undefined?

a. Cotangent $\frac{\cos}{\sin}$

b. Secant $\frac{1}{\cos}$

c. Tangent $\frac{\sin}{\cos}$

$[-2\pi, -\pi, 0, \pi, 2\pi]$

$[-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}]$

$[-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}]$

10. In which quadrants are the following trig functions positive?

a. $\tan \theta$
 I, III

b. $\sec \theta$
 I, IV

c. $\sin \theta$
 I, II



11. Evaluate the following:

a. $\sin(\tan^{-1}(1))$

b. $\arccos(\tan \frac{\pi}{4})$

c. $\tan^{-1}(\cos \pi)$

d. $\sin(\sin^{-1}(\frac{\pi}{6})) = \frac{\pi}{6}$

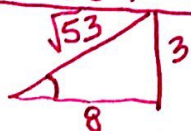
$\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$\arccos(1) = 0$

$\tan^{-1}(-1) = -\frac{\pi}{4}$

$\sin(\frac{\pi}{6}) = \frac{1}{2}$

12. Evaluate: $\cos(\tan^{-1}(\frac{3}{8}))$



$\cos \theta = \frac{8}{\sqrt{53}} = \boxed{\frac{8\sqrt{53}}{53}}$

13. Graph the following functions:

a. $f(x) = 2 \csc x$

b. $f(x) = -\sin(x - \frac{\pi}{2}) + 4$

c. $f(x) = 2 \cot x + 1$

on other sheet

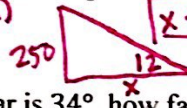
Calculator ACTIVE

14. Find the length of an arc created by a central angle of 135° with a radius of 7cm.

Arc length = $r\theta$, θ in radians $135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$

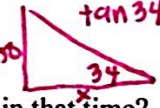
$S = 7 \left(\frac{3\pi}{4} \right) = 16.49$

15. From the top of a 250ft tall building, a woman observes a car moving toward the building. When she first sees the car, her angle of depression is 12° . $\tan 12 = 250/x$



$x = 1176.16 \text{ ft}$

$\tan 34 = \frac{x}{250}$



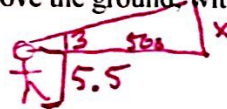
$1176.16 -$

370.64

$= 805.52$

- a. How far is the car from the building?
- b. If five seconds later, her angle of depression to the car is 34° , how far did the car travel in that time?

16. A hot air balloon is rising from ground level from a point 500ft away from the base of a photographer. If the photographer is holding his camera 5.5 feet above the ground, with an angle of elevation of 13° , how high off the ground is the hot air balloon?



$\tan 13 = \frac{x}{500}$

$x = 115.43 + 5.5 = 120.93$

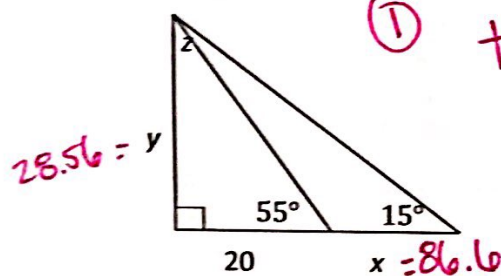
17. Convert degrees to radians or vice versa:

a. $25^\circ \cdot \frac{\pi}{180} = \frac{5\pi}{36}$

b. $\frac{\pi}{3} \cdot \frac{180}{\pi} = 60^\circ$

c. $-162^\circ \cdot \frac{\pi}{180} = -\frac{9\pi}{10}$

18. Solve for all of variables in the triangles below. (Note: z is the entire top angle.)



① $\tan(55) = \frac{y}{20}$

$y = 28.56$

② $\tan(15) = \frac{28.56}{x+20}$

$\frac{(x+20)\tan(15)}{\tan(15)} = \frac{28.56}{\tan(15)}$

$x+20 = 106.6$

$x = 86.6$

③ $\tan(z) = \frac{106.6}{28.56}$

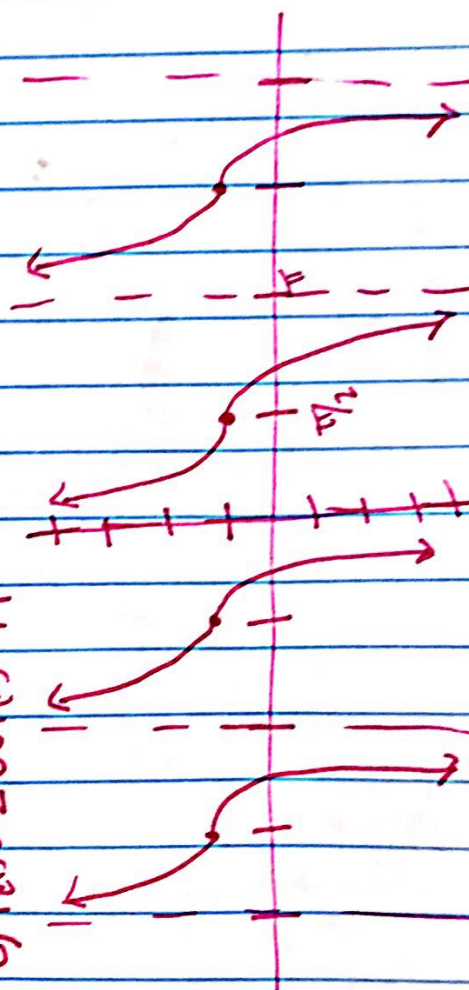
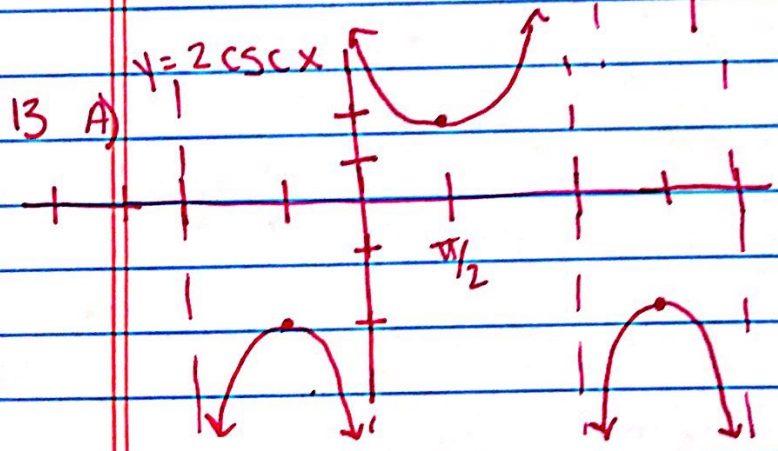
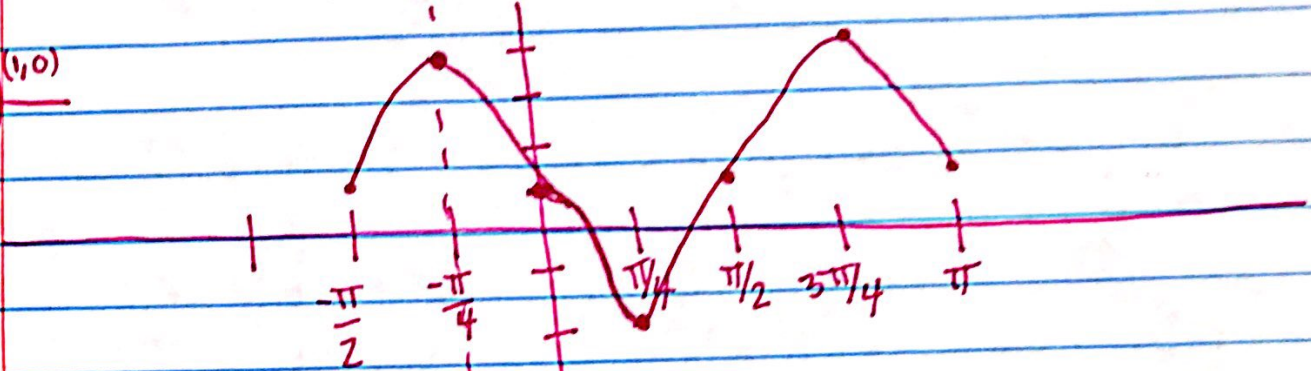
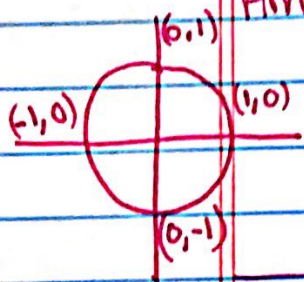
$\tan^{-1}\left(\frac{106.6}{28.56}\right) = 75^\circ$

7 $f(x) = 3 \cos(2(x + \pi/4)) + 1$

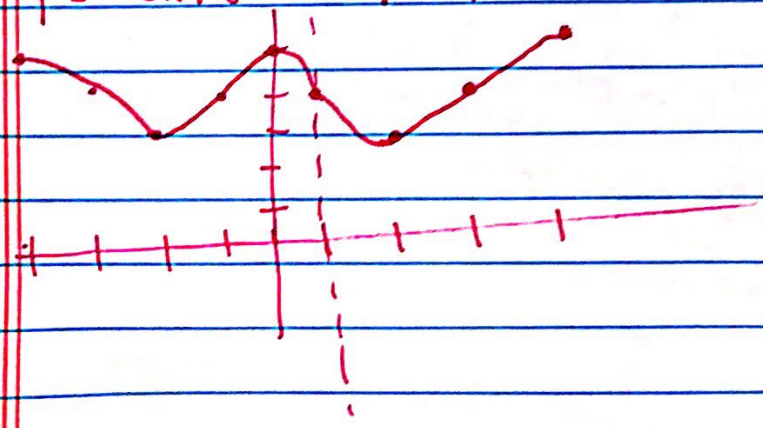
Amp: 3

period: π

Phase Shift: $\pi/4$ V.S. up 1



b) $y = -\sin(x - \pi/2) + 4$



c) $f(x) = 2 \cot(x) + 1$