SECTION 5.1 EXERCISES

In Exercises 1–4, evaluate without using a calculator. Use the Pythagorean identities rather than reference triangles. (See Example 1.)

1. Find $\sin \theta$ and $\cos \theta$ if $\tan \theta = 3/4$ and $\sin \theta > 0$.

2. Find $\sec \theta$ and $\csc \theta$ if $\tan \theta = 3$ and $\cos \theta > 0$.

3. Find $\tan \theta$ and $\cot \theta$ if $\sec \theta = 4$ and $\sin \theta < 0$.

4. Find $\sin \theta$ and $\tan \theta$ if $\cos \theta = 0.8$ and $\tan \theta < 0$.

In Exercises 5–8, use identities to find the value of the expression.

5. If $\sin \theta = 0.45$, find $\cos (\pi/2 - \theta)$.

6. If $\tan(\pi/2 - \theta) = -5.32$, find $\cot \theta$.

7. If $\sin(\theta - \pi/2) = 0.73$, find $\cos (-\theta)$.

8. If $\cot(-\theta) = 7.89$, find $\tan (\theta - \pi/2)$.

In Exercises 9–16, use basic identities to simplify the expression.

9. $\tan x \cos x$

10. $\cot x \tan x$

11. $\sec y \sin (\pi/2 - y)$

12. $\cot u \sin u$

13. $\dfrac{1 + \tan^2 x}{\csc^2 x}$

14. $\dfrac{1 - \cos^2 \theta}{\sin \theta}$

15. $\cos x - \cos^3 x$

16. $\dfrac{\sin^2 u + \tan^2 u + \cos^2 u}{\sec u}$

In Exercises 17–22, simplify the expression to either 1 or −1.

17. $\sin x \csc (-x)$

18. $\sec (-x) \cos (-x)$

19. $\cot (-x) \cot (\pi/2 - x)$

20. $\cot (-x) \tan (-x)$

21. $\sin^2 (-x) + \cos^2 (-x)$

22. $\sec^2 (-x) - \tan^2 x$

In Exercises 23–26, simplify the expression to either a constant or a basic trigonometric function. Support your result graphically.

23. $\dfrac{\tan (\pi/2 - x) \csc x}{\csc^2 x}$

24. $\dfrac{1 + \tan x}{1 + \cot x}$

25. $(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x)$

26. $\dfrac{\sec^2 u - \tan^2 u}{\cos^2 v + \sin^2 v}$

In Exercises 27–32, use the basic identities to change the expression to one involving only sines and cosines. Then simplify to a basic trigonometric function.

27. $(\sin x)(\tan x + \cot x)$

28. $\sin \theta - \tan \theta \cos \theta + \cos (\pi/2 - \theta)$

29. $\sin x \cos x \tan x \sec x \csc x$

30. $\dfrac{(\sec y - \tan y)(\sec y + \tan y)}{\sec y}$

31. $\dfrac{\tan x}{\csc^2 x} + \dfrac{\tan x}{\sec^2 x}$

32. $\dfrac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x}$

In Exercises 33–38, combine the fractions and simplify to a multiple of a power of a basic trigonometric function (e.g., $3 \tan^2 x$).

33. $\dfrac{1}{\sin^2 x} + \dfrac{\sec^2 x}{\tan^2 x}$

34. $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x}$

35. $\dfrac{\sin x}{\cot^2 x} - \dfrac{\sin x}{\cos^2 x}$

36. $\dfrac{1}{\sec x - 1} - \dfrac{1}{\sec x + 1}$

37. $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x}$

38. $\dfrac{\sin x}{1 - \cos x} + \dfrac{1 - \cos x}{\sin x}$

In Exercises 39–46, write each expression in factored form as an algebraic expression of a single trigonometric function (e.g., $(2 \sin x + 3)(\sin x - 1)$).

39. $\cos^2 x + 2 \cos x + 1$

40. $1 - 2 \sin x + \sin^2 x$

41. $1 - 2 \sin x + (1 - \cos^2 x)$

42. $\sin x - \cos^2 x - 1$

43. $\cos x - 2\, \widehat{\sin^2 x} + 1$

44. $\sin^2 x + \dfrac{2}{\csc x} + 1$

45. $4 \tan^2 x - \dfrac{4}{\cot x} + \sin x \csc x$

46. $\sec^2 x - \sec x + \tan^2 x$

In Exercises 47–50, write each expression as an algebraic expression of a single trigonometric function (e.g., $2 \sin x + 3$).

47. $\dfrac{1 - \sin^2 x}{1 + \sin x}$

48. $\dfrac{\tan^2 \alpha - 1}{1 + \tan \alpha}$

49. $\dfrac{\sin^2 x}{1 + \cos x}$

50. $\dfrac{\tan^2 x}{\sec x + 1}$

In Exercises 51–56, find all solutions to the equation in the interval $[0, 2\pi)$. You do not need a calculator.

51. $2 \cos x \sin x - \cos x = 0$

52. $\sqrt{2} \tan x \cos x - \tan x = 0$

53. $\tan x \sin^2 x = \tan x$

54. $\sin x \tan^2 x = \sin x$

55. $\tan^2 x = 3$

56. $2 \sin^2 x = 1$

In Exercises 57–62, find all solutions to the equation. You do not need a calculator.

57. $4 \cos^2 x - 4 \cos x + 1 = 0$ **58.** $2 \sin^2 x + 3 \sin x + 1 = 0$

59. $\sin^2 \theta - 2 \sin \theta = 0$ **60.** $3 \sin t = 2 \cos^2 t$

61. $\cos (\sin x) = 1$ **62.** $2 \sin^2 x + 3 \sin x = 2$

In Exercises 63–68, find all solutions to the trigonometric equation, using a calculator where needed.

63. $\cos x = 0.37$ **64.** $\cos x = 0.75$

65. $\sin x = 0.30$ **66.** $\tan x = 5$

67. $\cos^2 x = 0.4$ **68.** $\sin^2 x = 0.4$

In Exercises 69–74, make the suggested trigonometric substitution, and then use Pythagorean identities to write the resulting function as a multiple of a basic trigonometric function.

69. $\sqrt{1 - x^2}$, $x = \cos \theta$

70. $\sqrt{x^2 + 1}$, $x = \tan \theta$

71. $\sqrt{x^2 - 9}$, $x = 3 \sec \theta$

72. $\sqrt{36 - x^2}$, $x = 6 \sin \theta$

73. $\sqrt{x^2 + 81}$, $x = 9 \tan \theta$

74. $\sqrt{x^2 - 100}$, $x = 10 \sec \theta$

Standardized Test Questions

75. True or False If $\sec(x - \pi/2) = 34$, then $\csc x = 34$. Justify your answer.