SOLUTION Since $a \sin(bx + c) = a(\sin bx \cos c + \cos bx \sin c)$, we have

$$2 \sin x + 5 \cos x = a(\sin bx \cos c + \cos bx \sin c)$$
$$= (a \cos c) \sin bx + (a \sin c) \cos bx.$$

Comparing coefficients, we see that $b = 1$ and that $a \cos c = 2$ and $a \sin c = 5$.

We can solve for a as follows:

$$(a \cos c)^2 + (a \sin c)^2 = 2^2 + 5^2$$
$$a^2 \cos^2 c + a^2 \sin^2 c = 29$$
$$a^2(\cos^2 c + \sin^2 c) = 29$$
$$a^2 = 29 \quad \text{Pythogorean identity}$$
$$a = \pm\sqrt{29}$$

If we choose a to be positive, then $\cos c = 2/\sqrt{29}$ and $\sin c = 5/\sqrt{29}$. We can identify an acute angle c with those specifications as either $\cos^{-1}(2/\sqrt{29})$ or $\sin^{-1}(5/\sqrt{29})$, which are equal. So, an exact sinusoid for f is

$$f(x) = 2 \sin x + 5 \cos x$$
$$= a \sin(bx + c)$$
$$= \sqrt{29} \sin(x + \cos^{-1}(2/\sqrt{29})) \text{ or } \sqrt{29} \sin(x + \sin^{-1}(5/\sqrt{29}))$$

Now try Exercise 43.

QUICK REVIEW 5.3

(For help, go to Sections 4.2 and 5.1.)

In Exercises 1–6, express the angle as a sum or difference of special angles (multiples of 30°, 45°, $\pi/6$, or $\pi/4$). Answers are not unique.

1. 15°
2. 75°
3. 165°
4. $\pi/12$
5. $5\pi/12$
6. $7\pi/12$

In Exercises 7–10, tell whether or not the identity $f(x + y) = f(x) + f(y)$ holds for the function f.

7. $f(x) = \ln x$
8. $f(x) = e^x$
9. $f(x) = 32x$
10. $f(x) = x + 10$

SECTION 5.3 EXERCISES

In Exercises 1–10, use a sum or difference identity to find an exact value.

1. sin 15°
2. tan 15°
3. sin 75°
4. cos 75°
5. $\cos \frac{\pi}{12}$
6. $\sin \frac{7\pi}{12}$
7. $\tan \frac{5\pi}{12}$
8. $\tan \frac{11\pi}{12}$
9. $\cos \frac{7\pi}{12}$
10. $\sin \frac{-\pi}{12}$

In Exercises 11–22, write the expression as the sine, cosine, or tangent of an angle.

11. $\sin 42° \cos 17° - \cos 42° \sin 17°$
12. $\cos 94° \cos 18° + \sin 94° \sin 18°$
13. $\sin \frac{\pi}{5} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \cos \frac{\pi}{5}$
14. $\sin \frac{\pi}{3} \cos \frac{\pi}{7} - \sin \frac{\pi}{7} \cos \frac{\pi}{3}$
15. $\dfrac{\tan 19° + \tan 47°}{1 - \tan 19° \tan 47°}$

16. $\dfrac{\tan(\pi/5) - \tan(\pi/3)}{1 + \tan(\pi/5)\tan(\pi/3)}$

17. $\cos\dfrac{\pi}{7}\cos x + \sin\dfrac{\pi}{7}\sin x$ **18.** $\cos x\cos\dfrac{\pi}{7} - \sin x\sin\dfrac{\pi}{7}$

19. $\sin 3x\cos x - \cos 3x\sin x$

20. $\cos 7y\cos 3y - \sin 7y\sin 3y$

21. $\dfrac{\tan 2y + \tan 3x}{1 - \tan 2y\tan 3x}$

22. $\dfrac{\tan 3\alpha - \tan 2\beta}{1 + \tan 3\alpha\tan 2\beta}$

In Exercises 23–30, prove the identity.

23. $\sin\left(x - \dfrac{\pi}{2}\right) = -\cos x$ **24.** $\tan\left(x - \dfrac{\pi}{2}\right) = -\cot x$

25. $\cos\left(x - \dfrac{\pi}{2}\right) = \sin x$

26. $\cos\left[\left(\dfrac{\pi}{2} - x\right) - y\right] = \sin(x + y)$

27. $\sin\left(x + \dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}\sin x + \dfrac{1}{2}\cos x$

28. $\cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}(\cos x + \sin x)$

29. $\tan\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

30. $\cos\left(\theta + \dfrac{\pi}{2}\right) = -\sin\theta$

In Exercises 31–34, match each graph with a pair of the following equations. Use your knowledge of identities and transformations, not your grapher.

(a) $y = \cos(3 - 2x)$

(b) $y = \sin x\cos 1 + \cos x\sin 1$

(c) $y = \cos(x - 3)$

(d) $y = \sin(2x - 5)$

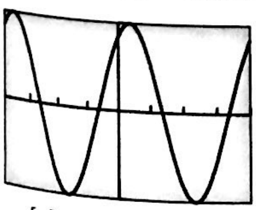
(e) $y = \cos x\cos 3 + \sin x\sin 3$

(f) $y = \sin(x + 1)$

(g) $y = \cos 3\cos 2x + \sin 3\sin 2x$

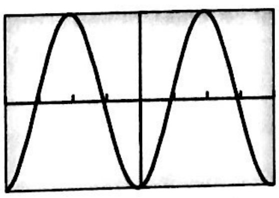
(h) $y = \sin 2x\cos 5 - \cos 2x\sin 5$

31.



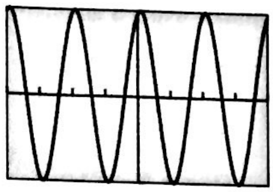
$[-2\pi, 2\pi]$ by $[-1, 1]$

32.



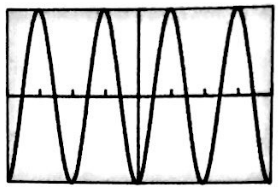
$[-2\pi, 2\pi]$ by $[-1, 1]$

33.



$[-2\pi, 2\pi]$ by $[-1, 1]$

34.



$[-2\pi, 2\pi]$ by $[-1, 1]$

In Exercises 35 and 36, use sum or difference identities (and not your grapher) to solve the equation exactly.

35. $\sin 2x\cos x = \cos 2x\sin x$ **36.** $\cos 3x\cos x = \sin 3x\sin x$

In Exercises 37–42, prove the reduction formula.

37. $\sin\left(\dfrac{\pi}{2} - u\right) = \cos u$ **38.** $\tan\left(\dfrac{\pi}{2} - u\right) = \cot u$

39. $\cot\left(\dfrac{\pi}{2} - u\right) = \tan u$ **40.** $\sec\left(\dfrac{\pi}{2} - u\right) = \csc u$

41. $\csc\left(\dfrac{\pi}{2} - u\right) = \sec u$ **42.** $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$

In Exercises 43–46, express the function as a sinusoid in the form $y = a\sin(bx + c)$.

43. $y = 3\sin x + 4\cos x$ **44.** $y = 5\sin x - 12\cos x$

45. $y = \cos 3x + 2\sin 3x$ **46.** $y = 3\cos 2x - 2\sin 2x$

In Exercises 47–55, prove the identity.

47. $\sin(x - y) + \sin(x + y) = 2\sin x\cos y$

48. $\cos(x - y) + \cos(x + y) = 2\cos x\cos y$

49. $\cos 3x = \cos^3 x - 3\sin^2 x\cos x$

50. $\sin 3u = 3\cos^2 u\sin u - \sin^3 u$

51. $\cos 3x + \cos x = 2\cos 2x\cos x$

52. $\sin 4x + \sin 2x = 2\sin 3x\cos x$

53. $\tan(x + y)\tan(x - y) = \dfrac{\tan^2 x - \tan^2 y}{1 - \tan^2 x\tan^2 y}$

54. $\tan 5u\tan 3u = \dfrac{\tan^2 4u - \tan^2 u}{1 - \tan^2 4u\tan^2 u}$

55. $\dfrac{\sin(x + y)}{\sin(x - y)} = \dfrac{(\tan x + \tan y)}{(\tan x - \tan y)}$

Standardized Test Questions

56. True or False If A and B are supplementary angles, then $\cos A + \cos B = 0$. Justify your answer.

57. True or False If $\cos A + \cos B = 0$, then A and B are supplementary angles. Justify your answer.