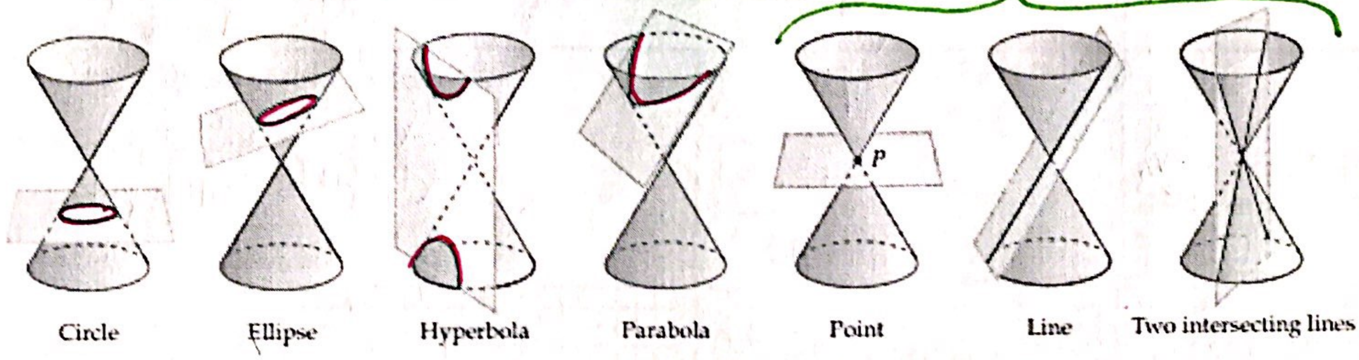
Day 1: Circles & Parabolas

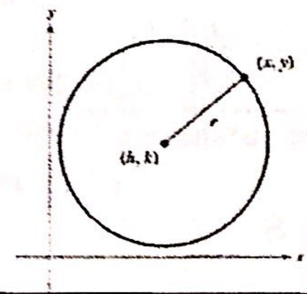
degenerate conics



Conic Section: the intersection of a plane and a double-napped cone

A circle is the set of all points (x, y) that are equidistant from a fixed point (h, k), called the center of the circle.

Info about Circles	
Standard Equation	$(x-h)^2 + (y-k)^2 = r^2$
Center	(h, k)
Length of Radius	r



➢ To *recognize* that the equation of a conic is a circle, notice that there are two squared terms with the same positive coefficients.

Write the equation of the circle in standard form.

1. $x^2 + y^2 - 10x + 8y = 23$	2. $16x^2 + 16y^2 + 8x - 32y - 127 = 0$
$x^2 - 10x \quad + y^2 + 8y \quad = 23$	$16x^2 + 8x \quad 16y^2 - 32y \quad = 127$
$(x^2 - 10x + 25) + (y^2 + 8y + 16) = 23 + 25 + 16$	$16(x^2 + \frac{1}{2}x + \frac{1}{16}) + 16(y^2 - 2y + 1) = 127 + 16(\frac{1}{16}) + 16(1)$
$\boxed{(x-5)^2 + (y+4)^2 = 64}$	$\frac{16(x + \frac{1}{4})^2 + 16(y-1)^2}{16} = \frac{144}{16}$
	$\boxed{(x + \frac{1}{4})^2 + (y-1)^2 = 9}$

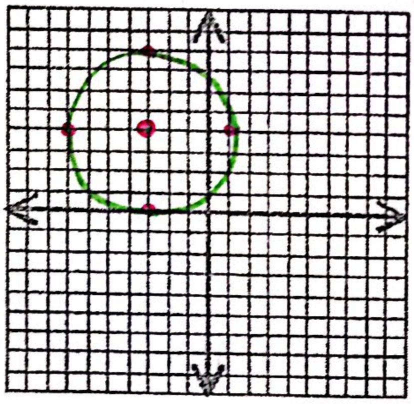
For each of the following:

- Write the equation in standard form, if necessary.
- Find the center and the exact length of the radius.
- Graph the circle.

3. $(x+3)^2+(y-4)^2=16$

center: $(-3,4)$

$r=4$



4. $x^2+y^2+2x+4y=9$

$x^2+2x \quad +y^2+4y \quad =9$

$(x^2+2x+\underline{1})+(y^2+4y+\underline{4})=9+\underline{1}+\underline{4}$

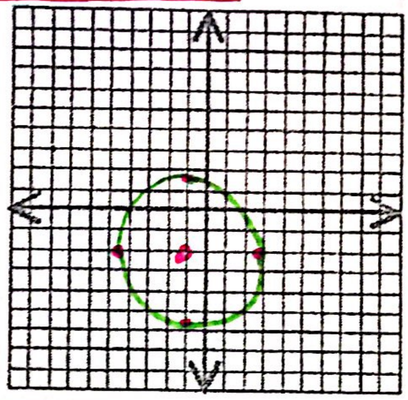
$\boxed{(x+1)^2+(y+2)^2=14}$

Center: $(-1,-2)$

$r^2=14$

$r=\sqrt{14}$

$r\approx 3.74$



Write the equation of the circle that satisfies each set of conditions.

5. The circle passes through $(7,-1)$ and has its center at $(-2,4)$.

x y

h, k

$(x+2)^2+(y-4)^2=r^2$

$(7+2)^2+(-1-4)^2=r^2$

$(9)^2+(-5)^2=r^2$

$81+25=r^2$

$106=r^2$

$\boxed{(x+2)^2+(y-4)^2=106}$

6. The endpoints of a diameter are at $(-2,-3)$ and $(4,5)$.

midpoint $\rightarrow \left(\frac{-2+4}{2}, \frac{-3+5}{2}\right)$

center: $(1,1)$

h, k

$(x-1)^2+(y-1)^2=r^2$

$(4-1)^2+(5-1)^2=r^2$

$(3)^2+(4)^2=r^2$

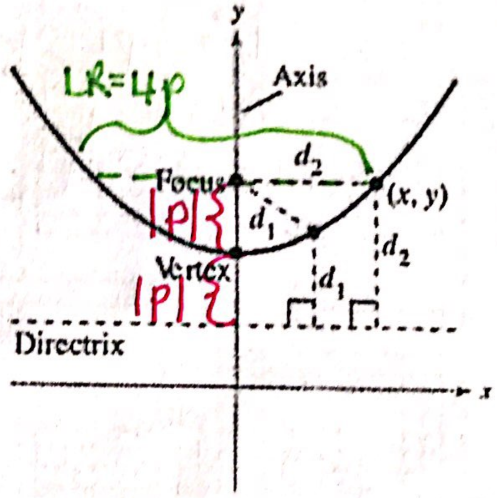
$25=r^2$

$\boxed{(x-1)^2+(y-1)^2=25}$

Parabola: The set of all points (x, y) in a plane that are equidistant from a fixed line called the directrix, and a fixed point called the focus.

Info about Parabolas

Standard Equation	$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Axis of Symmetry (AOS)	$x = h$	$y = k$
Vertex	(h,k)	(h,k)
Focus	$(h, k+p)$	$(h+p, k)$
Directrix	$y = k - p$	$x = h - p$
Direction of Opening	Upward if $p > 0$ Downward if $p < 0$	Right if $p > 0$ Left if $p < 0$
Latus Rectum (LR)	$4p$	$4p$



- The midpoint between the focus and the directrix is the vertex.
- The line passing through the focus and the vertex is the axis of symmetry.
- The directrix and the axis of symmetry are always perpendicular.
- The latus rectum is a line segment perpendicular to the axis of symmetry that passes through the focus and has endpoints on the parabola.
- To *recognize* that the equation of a conic is a parabola, notice that there is only one squared term.

Try to remember the following rules when it comes to standard form parabolas:

If you have an x^2, but no y^2 → vertical parabola.
If you have a y^2, but no x^2 → horizontal parabola.

Write the standard form of the equation for each parabola. Find and graph all of the requested information.

1. $y = x^2 - 12x + 30$

$y - 30 = x^2 - 12x$

$36 + y - 30 = (x^2 - 12x + 36)$

$y + 6 = (x-6)^2$

$\boxed{(x-6)^2 = 1(y+6)}$

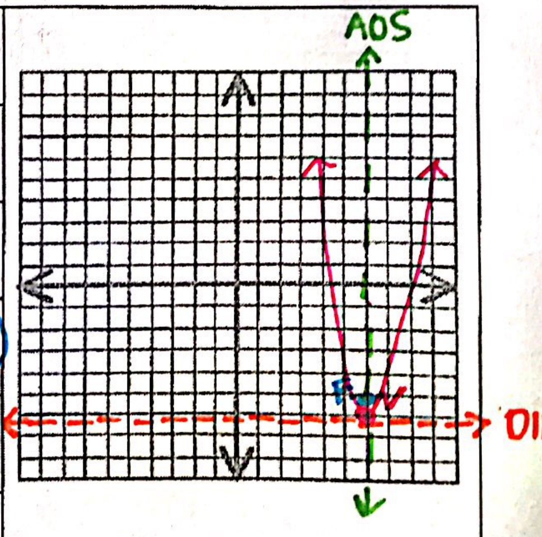
$4p = 1$

$p = 1/4$

Focus: $(6, -6 + 1/4)$

DIR: $y = -6 - 1/4$

Opens:	up
Vertex:	$(6, -6)$
AOS:	$x = 6$
Focus:	$(6, -5.75)$
Directrix:	$y = -6.25$
LR:	1



2. $y^2 - 4x + 2y + 5 = 0$

$y^2 + 2y = 4x - 5$

$(y^2 + 2y + \underline{1}) = 4x - 5 + \underline{1}$

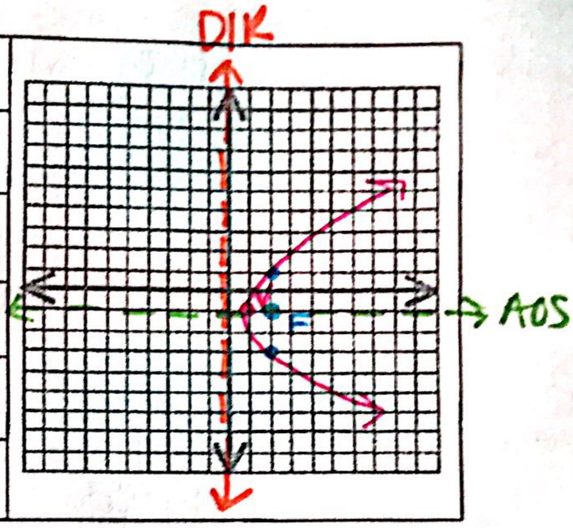
$(y+1)^2 = 4x - 4$

$(y+1)^2 = 4(x-1)$

Focus: $(1+1, -1)$ → $4p = 4$, $p = 1$

DIR: $x = 1 - 1$

Opens:	Right
Vertex:	$(1,-1)$
AOS:	$y = -1$
Focus:	$(2,-1)$
Directrix:	$x = 0$
LR:	4



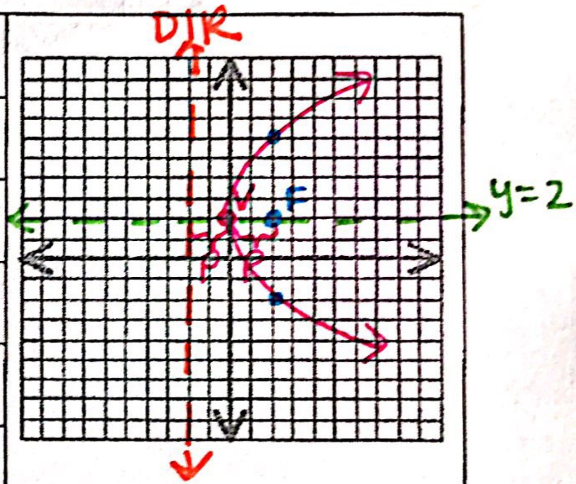
3. The focus is at $(2,2)$ and the equation of the directrix is $x = -2$.

$\boxed{p = 2}$

$(y-2)^2 = 8(x-0)$

$\boxed{(y-2)^2 = 8x}$

Opens:	Right
Vertex:	$(0,2)$
AOS:	$y = 2$
Focus:	$(2,2)$
Directrix:	$x = -2$
LR:	8



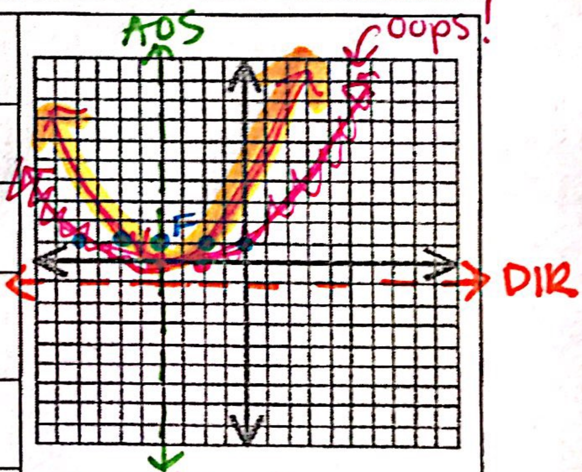
4. The vertex is at $(-2,0)$ and the coordinates of the focus are $\left(-2, \frac{1}{2}\right)$.

$p = \frac{1}{2}$

$(x+2)^2 = 2(y-0)$

$\boxed{(x+2)^2 = 2y}$

Opens:	up
Vertex:	$(-2,0)$
AOS:	$x = -2$
Focus:	$\left(-2, \frac{1}{2}\right)$
Directrix:	$y = -\frac{1}{2}$
LR:	$4p = 2$



*scale $= \frac{1}{2}$

Find the standard equation of the parabola; then find the coordinates of the vertex. Determine if the graph of the parabola will be a function.

5. $y^2 + 2y - x + 6 = 0$

$y^2 + 2y = x - 6$

$(y^2 + 2y + \underline{1}) = x - 6 + \underline{1}$

$(y+1)^2 = x - 5$

$\boxed{(y+1)^2 = 1(x-5)}$

vertex: $(5,-1)$

Not a function!