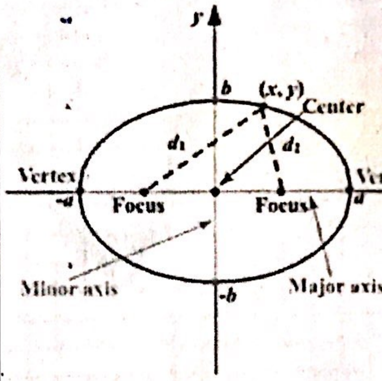
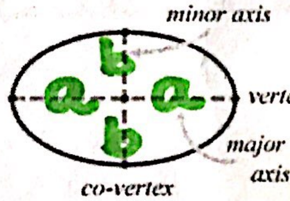
Day 2: Ellipses and Hyperbolas

Ellipse: the set of all points (x, y) in a plane, the sum of whose distances from two fixed points called foci is constant.



	Horizontal Major Axis	Vertical Major Axis
Standard Form:	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Center:	(h,k)	(h,k)
Vertices:	$(h \pm a, k)$	$(h, k \pm a)$
Covertices:	$(h, k \pm b)$	$(h \pm b, k)$
Foci:	$(h \pm c, k)$	$(h, k \pm c)$
Major Axis: length	Horizontal = $2a$	Vertical = $2a$
Minor Axis: length	Vertical = $2b$	Horizontal = $2b$
	$a^2=b^2+c^2$ and $a>b$ $a^2-b^2=c^2$	*a is always larger than b



- The line through the foci intersects the ellipse at two points called vertices.
- The chord joining these points is the major axis, and its midpoint is the center of the ellipse.
- The chord perpendicular to the major axis at the center is the minor axis.
- To recognize that the equation of a conic is an ellipse, notice that there are two quadratic terms with different coefficients with the same sign.

Examples:

1. Graph $4x^2+9y^2=36$

$$\frac{4x^2}{36}+\frac{9y^2}{36}=\frac{36}{36}$$

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

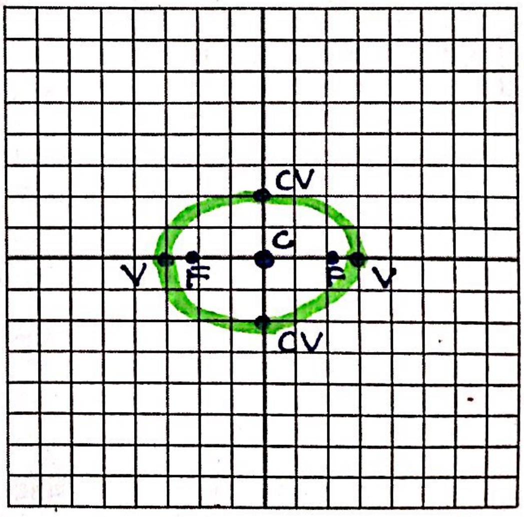
must always be equal to 1!

a=3 b=2

Center:	(0,0)
Vertices:	(3,0) (-3,0)
Covertices:	(0,2) (0,-2)
Foci:	$(\pm\sqrt{5},0)$
Major Axis:	2(3) = 6
Minor Axis:	2(2) = 4

$a^2-b^2=c^2$
$9-4=c^2$
$5=c^2$
$\pm\sqrt{5}=c$
≈ 2.2 for graphing

keep in this form



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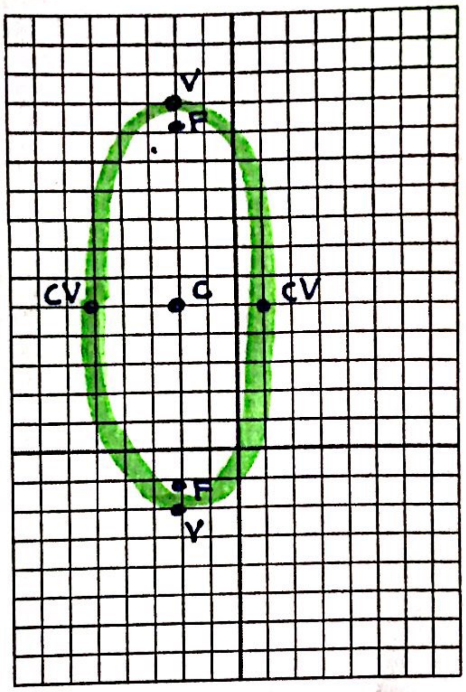
2. Graph $\frac{(x+2)^2}{9} + \frac{(y-5)^2}{49} = 1$

$b = 3$ $a = 7$

Center:	$(-2, 5)$
Vertices:	$(-2, 12)$ $(-2, -2)$
Covertices:	$(1, 5)$ $(-5, 5)$
Foci:	$(-2, 5 \pm 2\sqrt{10})$
Major Axis:	$2(7) = 14$
Minor Axis:	$2(3) = 6$

$a^2 - b^2 = c^2$
$49 - 9 = c^2$
$40 = c^2$
$\pm\sqrt{40} = c$
$\pm 2\sqrt{10} = c$
≈ 6.3



3. Graph: $x^2 + 4y^2 + 6x - 8y + 9 = 0$

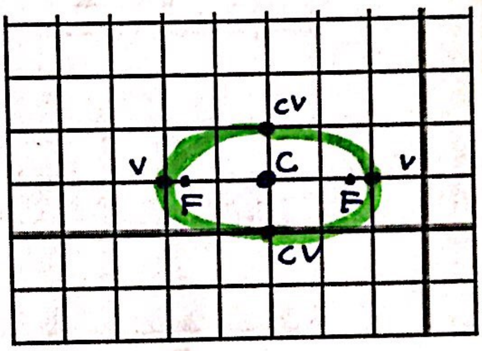
$x^2 + 6x \quad + 4y^2 - 8y = -9$
$(x^2 + 6x + \underline{9}) + 4(y^2 - 2y + \underline{1}) = -9 + \underline{9} + 4(\underline{1})$
$\frac{(x+3)^2}{4} + \frac{4(y-1)^2}{4} = \frac{4}{4}$

$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$

$a = 2$ $b = 1$

Center:	$(-3, 1)$
Vertices:	$(-5, 1)$ $(-1, 1)$
Covertices:	$(-3, 2)$ $(-3, 0)$
Foci:	$(-3 \pm \sqrt{3}, 1)$
Major Axis:	$2(2) = 4$
Minor Axis:	$2(1) = 2$

$4 - 1 = c^2$
$3 = c^2$
$\pm\sqrt{3} = c$
≈ 1.7



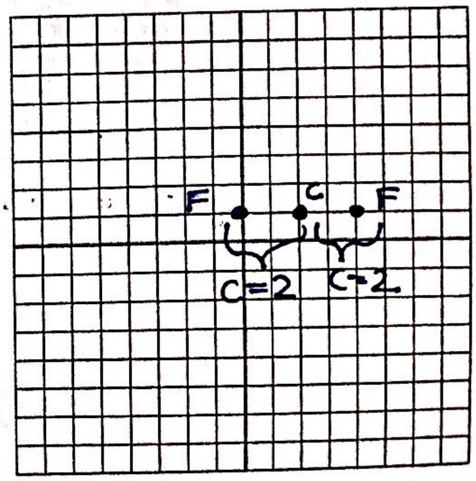
4. Find the standard form of the ellipse having foci at $(0,1)$ and $(4,1)$ and a major axis length of 6.

$\hookrightarrow 2a = 6$
$a = 3$

$\hookrightarrow$ center: $(2, 1)$
$\hookrightarrow x^2$
$\hookrightarrow c = 2$

$a^2 - b^2 = c^2$
$3^2 - b^2 = 2^2$
$9 - b^2 = 4$
$5 = b^2$

$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1$



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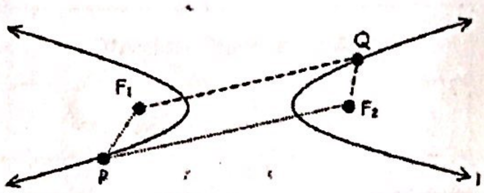
Hyperbolas:

Hyperbola: The set of all points (locus) in a plane such that the absolute value of the differences of the distance from the two foci is constant.

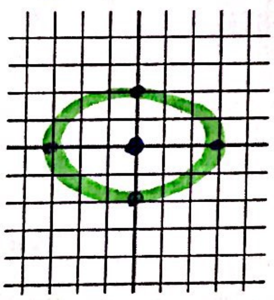


If you can graph an ellipse, you can graph a hyperbola!

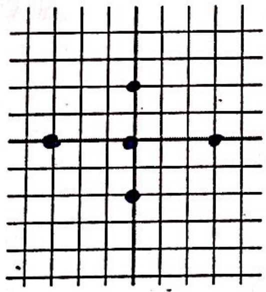
First, do a basic sketch of the given ellipse:

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

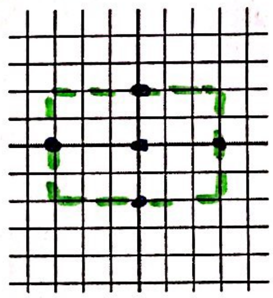
a=3 b=2



But instead of completely drawing the ellipse, let's just plot the main points:



Draw a rectangle using these points:

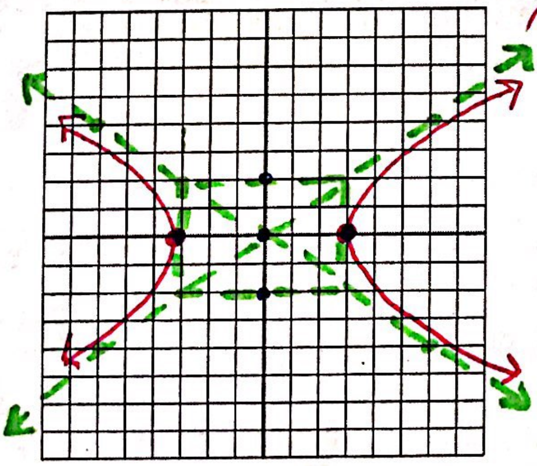


Now draw lines through the corners and graph the following hyperbola:

$$\frac{x^2}{9}-\frac{y^2}{4}=1$$

Notice the minus sign!

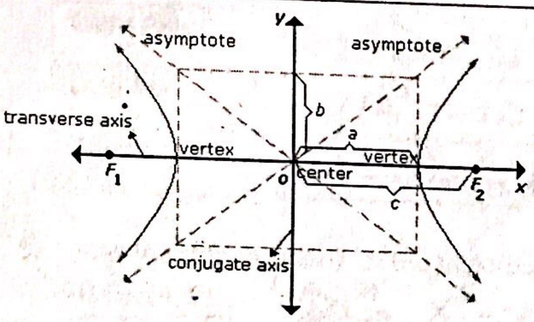
Since x is in front (positive) it's a sideways hyperbola!



these lines are asymptotes

*a is not necessarily the longest

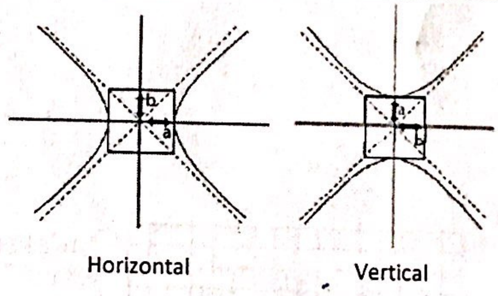
	Horizontal Transverse Axis	Vertical Transverse Axis
Standard Form:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center:	(h, k)	(h, k)
Vertices:	$(h \pm a, k)$	$(h, k \pm a)$
Foci:	$(h \pm c, k)$	$(h, k \pm c)$
Focal Axis: like AOS	$y = k$	$x = h$
Transverse Axis:	$2a$	$2a$
Conjugate Axis:	$2b$	$2b$
Asymptotes	$y = \pm \frac{b}{a}(x-h) + k$	$y = \pm \frac{a}{b}(x-h) + k$
	* $a^2 + b^2 = c^2$	



The following is true for both orientations:

The a-value is the distance from the centre to a vertex.

The b-value is the distance from the centre to a side of the box not attached to the hyperbola



- The center is the midpoint of the segment that would join the foci.
- The foci are c units from the center and the vertices are a units from the center.
- The transverse axis and the conjugate axis intersect at the center.
- To recognize that the equation of a conic is a hyperbola, notice that there are two quadratic terms with opposite signs (one positive, one negative).

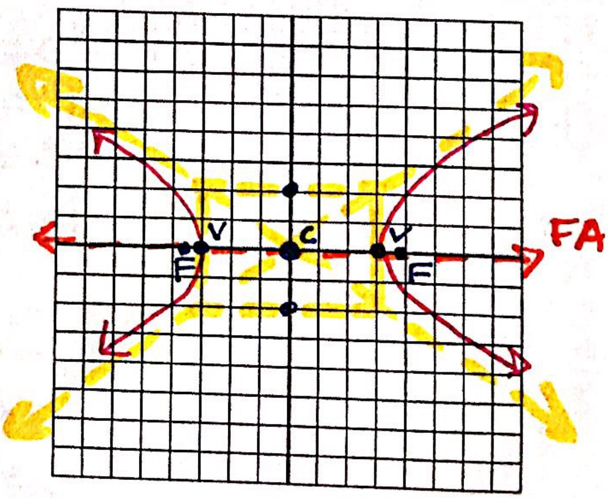
Example:

1. Graph $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$a=3 \quad b=2$

Center:	$(0,0)$
Vertices:	$(3,0)\ (-3,0)$
Foci:	$(\pm\sqrt{13}, 0)$
Focal Axis: *equation	$y=0$
Trans Axis: *length	$2(3) = 6$
Conj Axis: *length	$2(2) = 4$
Asymptotes:	$y = \pm 2/3\,(x-0)+0$ $\boxed{y = \pm 2/3\, x}$

$a^2+b^2=c^2$
$9+4=c^2$
$13=c^2$
$\pm\sqrt{13}=c$
≈ 3.6

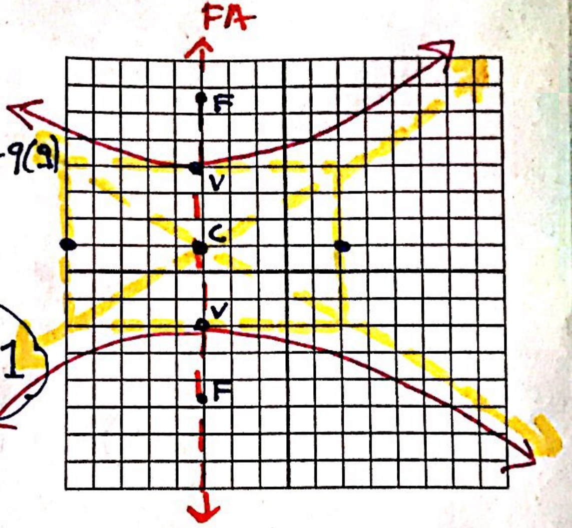


2. Graph $25y^2 - 9x^2 - 50y - 54x - 281 = 0$

$25y^2 - 50y \quad -9x^2 - 54x \quad = 281$

$25(y^2 - 2y + 1) - 9(x^2 + 6x + 9) = 281 + 25(1) - 9(9)$

$\dfrac{25(y-1)^2}{225} - \dfrac{9(x+3)^2}{225} = \dfrac{225}{225}$



Center:	$(-3, 1)$
Vertices:	$(-3,4)(-3,-2)$
Foci:	$(-3,\ 1 \pm \sqrt{34})$
Focal Axis:	$X = -3$
Trans Axis:	$2(3) = 6$
Conj Axis:	$2(5) = 10$
Asymptotes:	$y = \pm \frac{3}{5}(X+3) + 1$

$\dfrac{(y-1)^2}{9} - \dfrac{(x+3)^2}{25} = 1$

$a = 3 \quad b = 5$

$9 + 25 = c^2$

$34 = c^2$

$\pm\sqrt{34} = c$

≈ 5.8

3. Find the standard form of the hyperbola with foci $(-1, 2), (5, 2)$ and vertices at $(0, 2), (4, 2)$.

$x^2 - y^2$

center: $(2, 2)$

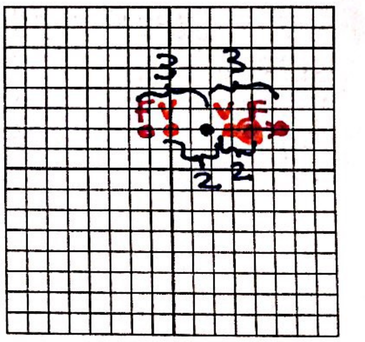
$c = 3 \quad a = 2$

$a^2 + b^2 = c^2$

$4 + b^2 = 9$

$b^2 = 5$

$\dfrac{(x-2)^2}{4} - \dfrac{(y-2)^2}{5} = 1$



4. Find the standard form of the hyperbola with foci at $(2, 5), (-4, 5)$ and transverse axis is 4 units long.

$2a = 4$

$a = 2$

$6 \to c = 3$

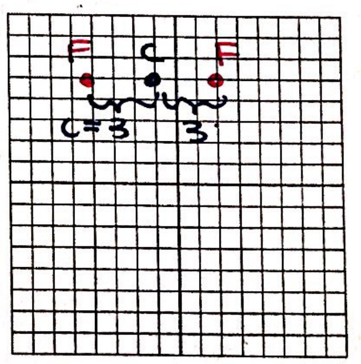
Center: $(-1, 5)$

$\dfrac{(x+1)^2}{4} - \dfrac{(y-5)^2}{5} = 1$

$a^2 + b^2 = c^2$

$4 + b^2 = 9$

$b^2 = 5$



5. Find the standard form of the hyperbola with vertices at $(3, -5), (3, 1)$ and asymptotes of $y = 2x - 8$ and $y = -2x + 4$

$m = \pm 2$

Center: $(3, -2)$

$6 \to a = 3$

$2 = \dfrac{\text{rise}}{\text{run}}$

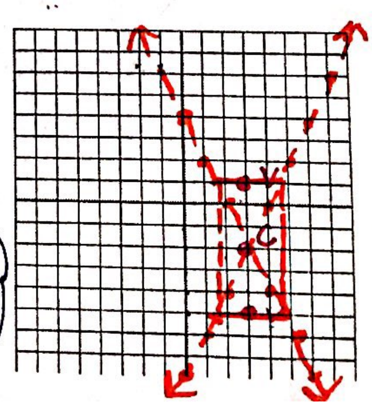
$2b = 3$

$2 = \dfrac{3}{b}$

$b = 3/2$

$\hookrightarrow b^2 = 9/4$

$\dfrac{(y+2)^2}{9} - \dfrac{(x-3)^2}{9/4} = 1$



Honors Precalculus

Name: Key 2015

Homework - Hyperbolas

Unit 10 Day 3

Graph the hyperbola and identify the center, vertices, foci, transverse axis, conjugate axis, and asymptotes.

1. $\frac{x^2}{16} - \frac{y^2}{25} = 1$

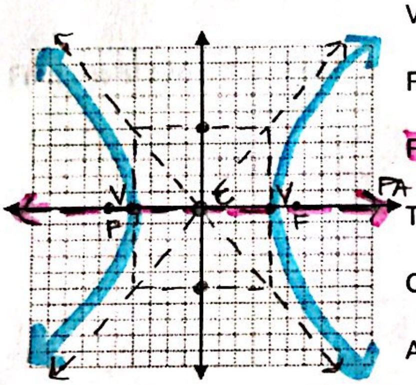
$a=4 \quad b=5$

$a^2+b^2=c^2$

$16+25=c^2$

$41=c^2$

$6.4 \approx \pm\sqrt{41} = c$



Center: $(0,0)$

Vertices: $(\pm 5, 0)$

Foci: $(\pm\sqrt{41}, 0)$

Foci Axis: $y=0$

Trans Axis: $2(4)=8$

Conj Axis: $2(5)=10$

Asymptotes:

$y = \pm \frac{5}{4}(x-0)+0$

$\boxed{y=\pm 5/4}$

2. $y^2 - 25x^2 = 100$

$\frac{y^2}{100} - \frac{25x^2}{100} = \frac{100}{100}$

$\frac{y^2}{100} - \frac{x^2}{4} = 1$

$a=10 \quad b=2$

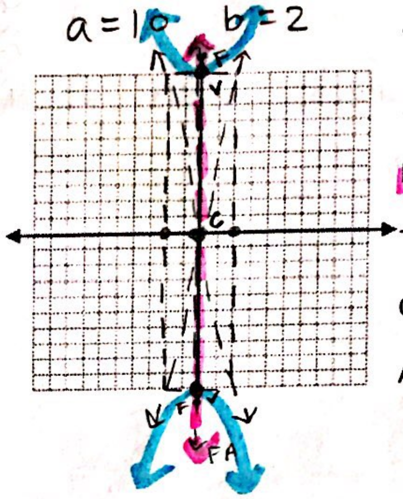
$100+4=c^2$

$104=c^2$

$\pm\sqrt{104}=c$

$\pm 2\sqrt{26}=c$

≈ 10.2



Center: $(0,0)$

Vertices: $(0, \pm 10)$

Foci: $(0, \pm 2\sqrt{26})$

Foci Axis: $x=0$

Trans Axis: $2(10)=20$

Conj Axis: $2(2)=4$

Asymptotes:

$y=\pm \frac{10}{2}(x-0)+0$

$\boxed{y=\pm 5x}$

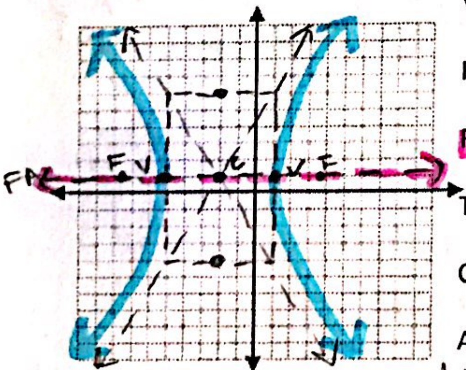
3. $\frac{(x+2)^2}{9} - \frac{(y-1)^2}{25} = 1$

$a=3 \quad b=5$

$9+25=c^2$

$34=c^2$

$\approx 5.8 \quad \pm\sqrt{34}=c$



Center: $(-2,1)$

Vertices: $(-5,1)(1,1)$

Foci: $(-5\pm\sqrt{34}, 1)$

Foci Axis: $y=1$

Trans Axis: $2(3)=6$

Conj Axis: $2(5)=10$

Asymptotes:

$\boxed{y=\pm \frac{5}{3}(x+2)+1}$

$(x-h)+k$

4. $\frac{y^2}{36} - \frac{(x+2)^2}{9} = 1$

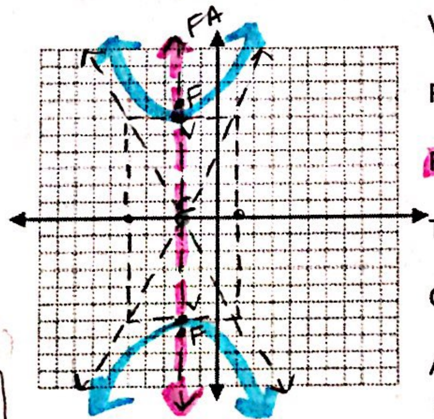
$a=6 \quad b=3$

$36+9=c^2$

$45=c^2$

$\pm\sqrt{45}=c$

$\approx 6.7 \quad \pm 3\sqrt{5}=c$



Center: $(-2,0)$

Vertices: $(-2,6)(-2,-6)$

Foci: $(-2, \pm 3\sqrt{5})$

Foci Axis: $x=-2$

Trans Axis: $2(6)=12$

Conj Axis: $2(3)=6$

Asymptotes:

$y=\pm \frac{6}{3}(x+2)+0$

$\boxed{y=\pm 2(x+2)}$