Thms

Factor Thm: $p(a)=0$ if and only if $x-a$ is a factor of $p(x)$

* if you divide a polynomial by $x-a$ and get zero as a remainder, then, not only is $x=a$ a zero of the polynomial, but $x-a$ is also a factor of the polynomial

ex Determine whether the given binomial is a factor of x^3+4x^2+x-6

a) $x+1$

$a=-1$

$(-1)^3+4(-1)^2+-1-6$

$-1+4-1-6=-4$

$x+1$ is not a factor

b) $x+3$

$a=-3$

$-3 \mid 1 \quad 4 \quad 1 \quad -6$

$\quad\quad 1 \quad -3 \quad -3 \quad 6$

$1 \quad 1 \quad -2 \quad 0$ ↑

$x+3$ is a factor

ex completely factor $y=x^3+2x^2-5x-6$ if $x+1$ is a factor

$-1 \mid 1 \quad 2 \quad -5 \quad -6$

$\quad\quad -1 \quad -1 \quad 6$

$1 \quad 1 \quad -6 \quad 0$

$(x+1)(x^2+x-6)$

$-\frac{6}{2}/3$

$(x+1)(x-2)(x+3)$

If need another: same question with $y=x^3-2x^2-5x+6$ $(x+2)$

Remainder Thm: The value of the polynomial $p(x)$ at $x=a$ is the same as the remainder you get when you divide that polynomial $p(x)$ by $x-a$

* to evaluate a polynomial $p(x)$ at $x=a$ use synthetic division to divide the polynomial by $x=a$. The remainder is $P(a)$

ex use synthetic division to find $P(3)$ for $P(x)=x^4-2x^3+x-9$

$$\begin{array}{c|ccccc} 3 & 1 & -2 & 0 & 1 & -9 \\ & & 3 & 3 & 9 & 30 \\ \hline & 1 & 1 & 3 & 10 & \boxed{21} \end{array} \leftarrow P(3)$$

$3^4-2(3)^3+3-9=21$

ex Use SD to find $P(-1)$ for $P(x)=2x^4+6x^3-5x^2-60$

ex $P(x)=3x^5-x^4-5x+10$ find $P(-2)$