$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

Math 3 Name Key

Unit 4 Day 2 Notes: Solving Polynomials by Factoring

Main Ideas/Questions	Notes/Examples
SOLVING POLYNOMIAL EQUATIONS *By Factoring*	① **MOVE ALL TERMS** to one side and set the equation **EQUAL TO 0.**
	② **FACTOR** the polynomial **completely!**
	③ **SET EACH FACTOR EQUAL TO 0** and **SOLVE** for each x-value.
	➢ For quadratic equations, solve by **square roots, completing the square,** or the **quadratic formula.**
	➢ **SIMPLY** all **irrational** and **complex** solutions!

Directions: Solve each equation by factoring.

1. $x^3+x=0$

$x(x^2+1)=0$

$x=0$

$x^2+1=0$, $-1\ -1$

$\sqrt{x^2}=\sqrt{-1}\rightarrow i\sqrt{1}$

$x=\pm i$

2. $4x^3-7x^2=3x^2$, $-3x^2\ -3x^2$

$4x^3-10x^2=0$

$2x^2(2x-5)=0$

$\frac{2x^2}{2}=\frac{0}{2}$

$x^2=0$

$x=0$ (multiplicity 2) (mult.)

$2x-5=0$, $+5\ +5$

$\frac{2x}{2}=\frac{5}{2}$

$x=\frac{5}{2}$

3. $x^4-\sqrt{64}=0$

$(x^2+8)(x^2-8)=0$

$x^2+8=0$, $-8\ -8$

$\sqrt{x^2}=\sqrt{-8}\rightarrow i\sqrt{8}$

$x=\pm 2i\sqrt{2}$

$x^2-8=0$, $+8\ +8$

$\sqrt{x^2}=\sqrt{8}$

$x=\pm 2\sqrt{2}$

4. $5x^5-80x=0$

$5x(x^4-16)=0$

$5x(x^2+4)(x^2-4)=0$

$5x(x^2+4)(x+2)(x-2)=0$

$5x=0$ $x+2=0$ $x-2=0$ $x^2+4=0$, $-4\ -4$

$x=0$ $x=-2$ $x=2$ $\sqrt{x^2}=\sqrt{-4}$

$x=\pm 2$ or $x=\pm 2i$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

5. $\sqrt[3]{x^3}+\sqrt[3]{125}=0$

$a=x \quad b=5$

$(x+5)(x^2-5x+25)=0$

$x+5=0$ $\quad x^2-5x+25=0$

$x=-5$ (Use Quadratic formula)

$a:1 \quad b^2-4ac$

$b:-5 \quad (-5)^2-4(1)(25)$

$c:25 \quad \sqrt{-75}=i\sqrt{75}=\pm 5i\sqrt{3}$

$x=\frac{5\pm 5i\sqrt{3}}{2}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

6. $8x^5+5x^2=4x^2$

$-4x^2 \quad -4x^2$

$8x^5+x^2=0$

$\sqrt[3]{8x^3} \quad \sqrt[3]{1}$

$a=2x \quad b=1$

$x^2(8x^3+1)=0$

$x^2(2x+1)(4x^2-2x+1)=0$

$x^2=0$ $\quad 2x+1=0$ $\quad 4x^2-2x+1=0$

$x=0$ (mult. 2) $\quad x=\frac{-1}{2}$

$a:4 \quad b^2-4ac$

$b:-2 \quad (-2)^2-4(4)(1)$

$c:1 \quad \sqrt{-12}=i\sqrt{12}=\pm 2i\sqrt{3}$

$x=\frac{2\pm 2i\sqrt{3}}{8}=\frac{1\pm i\sqrt{3}}{4}$

GCF: 2

7. $\sqrt[3]{x^3}-\sqrt[3]{216}=0$

$a=x \quad b=6$

$(x-6)(x^2+6x+36)=0$

$x-6=0$ $\quad x^2+6x+36=0$

$x=6$

$a:1 \quad b^2-4ac$

$b:6 \quad (6)^2-4(1)(36)$

$c:36 \quad \sqrt{-108}=i\sqrt{108}=\pm 6i\sqrt{3}$

$x=\frac{-6\pm 6i\sqrt{3}}{2}$

$x=-3\pm 3\sqrt{3}$

8. $16x^3=54$

$-54 \quad -54$

$16x^3-54=0$

$\sqrt[3]{8x^3} \quad \sqrt[3]{27}$

$a=2x \quad b=3$

$2(8x^3-27)=0$

$(2x-3)(4x^2+6x+9)=0$

$2x-3=0$ $\quad 4x^2+6x+9=0$

$x=\frac{3}{2}$

$a:4 \quad b^2-4ac$

$b:6 \quad (6)^2-4(4)(9)$

$c:9 \quad \sqrt{-108}=\pm 6i\sqrt{3}$

$x=\frac{-6\pm 6i\sqrt{3}}{8}=\frac{-3\pm 3i\sqrt{3}}{4}$

9. $x^4+x^2-42=0$

-42 / $-6 \quad 7$ / 1

-42 / $7 \quad -6$

$(x^2-6)(x^2+7)=0$

$x^2+6=0$ $\quad x^2+7=0$

$+6 \quad +6 \quad -7 \quad -7$

$\sqrt{x^2}=\sqrt{6} \quad \sqrt{x^2}=\sqrt{-7}=i\sqrt{7}$

$x=\pm\sqrt{6} \quad x=\pm i\sqrt{7}$

10. $x^4+3=13-9x^4$

$+9x^4-13 \quad -13+9x^4$

$10x^4-10=0$

$10(x^4-1)=0$

$(x^2+1)(x^2-1)=0$

$(x^2+1)(x+1)(x-1)=0$

$x^2+1=0$ $\quad x+1=0$ $\quad x-1=0$

$-1 \quad -1 \quad x=-1 \quad x=1$

$\sqrt{x^2}=\sqrt{-1}=i\sqrt{1} \quad x=\pm 1$

$x=\pm i$

11. $x^3 - 8x^2 + 16x = 0$

$x(x^2 - 8x + 16) = 0$

16, -4×-4, -8

$x(x-4)(x-4) = 0$

or

$x(x-4)^2 = 0$

$x = 0$ $\quad x - 4 = 0$

$x = 4$

(mult. 2)

12. $2x^6 - 10x^4 - 48x^2 = 0$

$2x^2(x^4 - 5x^2 - 24) = 0$

-24, 3×-8, -5

$2x^2(x^2 + 3)(x^2 - 8) = 0$

$\frac{2x^2}{2} = \frac{0}{2}$ $\quad x^2 + 3 = 0$ $\quad x^2 - 8 = 0$

$x^2 = 0$ $\quad \sqrt{x^2} = \sqrt{-3} = i\sqrt{3}$ $\quad \sqrt{x^2} = \sqrt{8} = 2\sqrt{2}$

$x = 0$ $\quad x = \pm i\sqrt{3}$ $\quad x = \pm 2\sqrt{2}$

(mult. 2)

13. $3x^4 - 2x^2 = 16$

$-16 \quad -16$

$3x^4 - 2x^2 - 16 = 0$

-48, -8×6, -2

$-\frac{8}{3}$ $\quad \frac{6}{3} = 2$

$(x^2 - \frac{8}{3})(x^2 + 2) = 0$

$(3x^2 - 8)(x^2 + 2) = 0$

$3x^2 + 8 = 0$ $\quad x^2 + 2 = 0$

$\frac{3x^2}{3} = \frac{8}{3}$ $\quad \sqrt{x^2} = \sqrt{-2} \to i\sqrt{2}$

$\sqrt{x^2} = \sqrt{\frac{8}{3}}$ $\quad x = \pm\sqrt{\frac{8}{3}} = \frac{2\sqrt{6}}{3}$ $\quad x = \pm i\sqrt{2}$

14. $5x^4 + 13x^2 - 8 = x^2 + 1$

$-x^2 - 1 \quad -x^2 - 1$

$5x^4 + 12x^2 - 9 = 0$

-45, 15×-3, 12

$\frac{15}{5} = 3$ $\quad \frac{-3}{5}$

$(x^2 + 3)(5x^2 - 3) = 0$

$x^2 + 3 = 0$ $\quad 5x^2 + 3 = 0$

$\sqrt{x^2} = \sqrt{-3} = i\sqrt{3}$ $\quad \frac{5x^2}{5} = \frac{-3}{5}$

$x = \pm i\sqrt{3}$ $\quad \sqrt{x^2} = \sqrt{\frac{3}{5}}$

$x = \pm\sqrt{\frac{3}{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

15. $(x^3 + 2x^2) + (5x + 10) = 0$

$x^2(x+2) + 5(x+2) = 0$

$(x^2 + 5)(x + 2) = 0$

$x^2 + 5 = 0$ $\quad x + 2 = 0$

$\sqrt{x^2} = \sqrt{-5} = i\sqrt{5}$ $\quad x = -2$

$x = \pm i\sqrt{5}$

16. $2x^3 - 3x^2 - 32x + 48 = 0$

$x^2(2x - 3) - 16(2x - 3) = 0$

$(x^2 - 16)(2x - 3) = 0$

$(x + 4)(x - 4)(2x - 3) = 0$

$x = -4$ $\quad x = 4$ $\quad x = \frac{3}{2}$

1c) $x^3 + 6x - 7 = 0$

What can we do?

look at 1st and last term

find factors possible $\pm 1, \pm 7$

try one to see if it a solution

$1^3 + 6(1) - 7 = 0$ ✓

$x = 1$ ← now use synthetic

1	1	0	6	−7
		1	1	7
	1	1	7	0

if $x = 1$ factor is $(x-1)$

$x^2 + x + 7 = 0$

$(x+1)(x^2 + x + 7) = 0$

$a = 1 \quad b = 1 \quad c = 7$

$$\frac{-1 \pm \sqrt{1 - 4(1)(7)}}{2(1)} = \frac{-1 \pm \sqrt{-27}}{2}$$

$$\frac{-1 \pm \sqrt{-9 \cdot 3}}{2} = \boxed{\frac{-1 \pm 3i\sqrt{3}}{2}} \quad \boxed{x = 1}$$

ex $2x^2 - x^2 - 18x + 9$

(2) $x^4 + x^3 - x^2 + x - 2 = 0$; $\pm 1, \pm 2$

$1^4 + 1^3 - 1^2 + 1 - 2 = 0$ ✓

1	1	1	−1	1	−2
		1	2	1	2
1	1	2	1	2	0

$(x^3 + 2x^2) + (x + 2) = 0$

$x^2(x+2) + 1(x+2) = 0$

$(x-1)(x^2+1)(x+2) = 0$

$x = 1$ $\quad x = -2$

$x^2 = -1$

$x = \pm i$

(3) $x^4 - 3x^3 + 5x - 3 = 0$; $\pm 1, \pm 3$

$1^4 - 3(1)^3 + 5(1) - 3 = 0$ ✓

1	1	−3	0	5	−3
		1	−2	−2	3
1	1	−2	−2	3	0

$(x-1)(x-1)$

$(x-1)(x-1)(x^2 - x - 3) = 0$

$(x-1)(x-1)$ (use quadr)

↓

keep going

$x^3 - 2x^2 - 2x + 3$

$1^3 - 2(1)^2 - 2(1) + 3 = 0$ ✓

1	1	−2	−2	3
		1	−1	3
1	1	−1	−3	0

Answer $\frac{1 \pm \sqrt{13}}{2}$, $x = 1$ mult 2