Graphing Rational Expressions

Example 1: Simplify the following. State any restrictions on the variables.

a) $\frac{(x+1)(x-5)}{(x-5)(x^2-1)} = \frac{1}{x-1}$

$(x-1)(x+1)$

$x \neq 5, 1, -1$

b) $\frac{x^2+x-12}{x^2+7x+12}$ $\frac{(x+4)(x-3)}{(x+4)(x+3)} = \frac{x-3}{x+3}$

$x \neq -4, -3$

Vertical Asymptotes: Where the Denominator of a function equals zero.

Point of Discontinuity: A hole in the graph.

Example 2: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2-1}{x^2-6x+5}$.

$\frac{(x-1)(x+1)}{(x-5)(x-1)}$ V.A: $x=5$

hole at $(1, \frac{2}{-4})$ $(1, -\frac{1}{2})$

Example 3: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{x^2-4}{x^2+5x+6}$.

VA: $x=-3$

hole: $(-2, -4)$

$\frac{(x-2)(x+2)}{(x+2)(x+3)}$

Horizontal Asymptotes: determined by comparing the degree of the numerator to the degree of the denominator. Let ***m*** = degree of numerator and ***n*** = degree of denominator.

If...	Then the graph has...
m < n $f(x) = \frac{x+4}{x^2+5x+4}$ $(x+4)(x+1)$	A horizontal asymptote at y = 0 V.A.: $x=-1$ Hole(s): $(-4, \frac{1}{-3})$ H.A.: $y=0$ Domain: $x \neq -4, -1$
m = n $(x+4)(x+1)$ $f(x) = \frac{x^2+5x+4}{4x^2-9}$ $(2x-3)(2x+3)$	A horizontal asymptote at the coefficient of m divided by the coefficient of n V.A.: $x=3/2, x=-3/2$ Hole(s): ____ H.A.: $y=1/4$ Domain: $x \neq 3/2, -3/2$
m > n $(x+4)(x+1)$ $f(x) = \frac{x^2+5x+4}{x+4}$	No horizontal asymptote V.A.: N/A Hole(s): $(-4, -3)$ H.A.: N/A Domain: $x \neq -4$

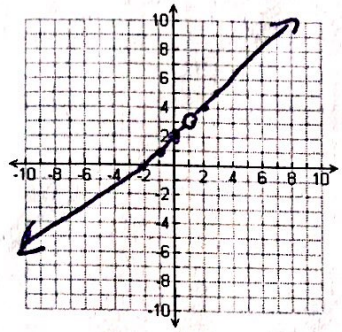
Example 4: State the asymptotes and points of discontinuity of each equation, and then graph the function and state the domain.

a) $f(x)=\dfrac{x^2+x-2}{x-1}$

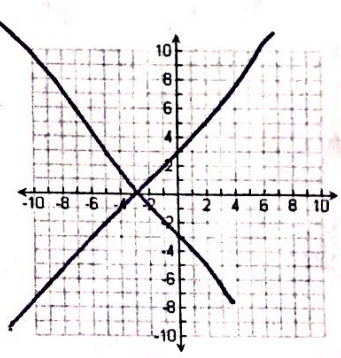
$\dfrac{(x-1)(x+2)}{x-1}$

hole: $(1,3)$

$x \neq 1$



b) $f(x)=\dfrac{2x^2+3}{x+2}$



c) $f(x)=\dfrac{x-1}{x^2-1}$

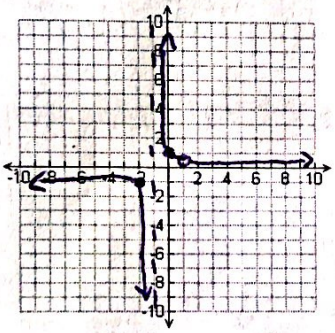
$\dfrac{x-1}{(x-1)(x+1)}$

hole: $(1,\frac{1}{2})$

V.A. $x=-1$ H.A. $y=0$

Intercepts: $f(0)=\dfrac{-1}{-1}=1$ $(0,1)$

$f(-2)=\dfrac{-2-1}{4-1}=\dfrac{-3}{3}=-1$ $(-2,1)$



d) $f(x)=\dfrac{x-3}{x^2-7x+12}$

$(x-3)(x-4)$

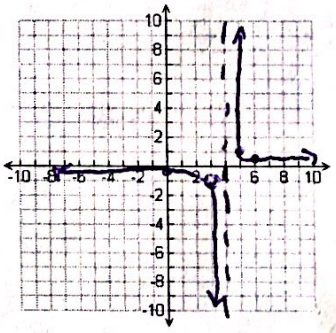
hole: $(3,-1)$

V.A. $x=4$

H.A $y=0$

$f(0)=\dfrac{-3}{12}=\dfrac{-1}{4}$ $(0,-\frac{1}{4})$

$f(6)=\dfrac{3}{3\cdot 2}=\dfrac{3}{6}=\dfrac{1}{2}$ $(6,\frac{1}{2})$



e) $f(x)=\dfrac{x^2+10x+25}{x^2+9x+20}$

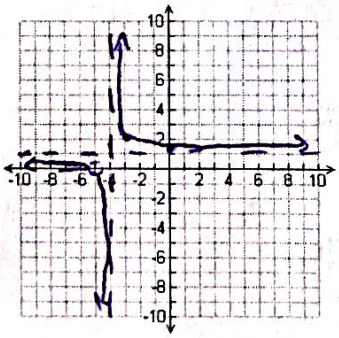
$\dfrac{(x+5)(x+5)}{(x+4)(x+5)}$

holes: $(-5,0)$

V.A: $x=-4$

H.A: $y=1$

$f(0)=\dfrac{25}{20}=\dfrac{5}{4}$ $(0,5/4)$ $f(-3)=\dfrac{4}{1\cdot 2}=\dfrac{9}{6}$ $(-3,2)$



f) $f(x)=\dfrac{x^2+12x+36}{x^2-36}$

$\dfrac{(x+6)(x+6)}{(x-6)(x+6)}$

holes: $(-6,0)$

H.A. $y=1$

V.A. $x=6$

$f(0)=\dfrac{36}{-36}=-1$

$f(4)=\dfrac{10}{-2}=-5$

$f(8)=\dfrac{14}{2}=7$

$f(10)=\dfrac{16}{4}=4$