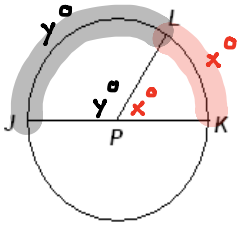
Math 3 Name Key

Unit 5 Day 3 Notes – Arcs and Angles

Part 1: Central Angles

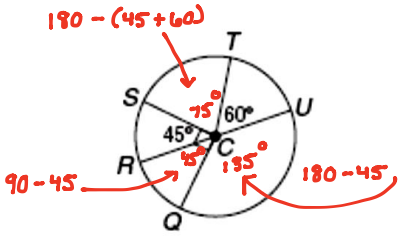
A central angle is an angle whose vertex is the center of the circle and whose other two points lie on the circle.
$\angle LPK$ and $\angle JPL$ are central angles in circle P.



Measure of a central angle = Measure of its intercepted arc

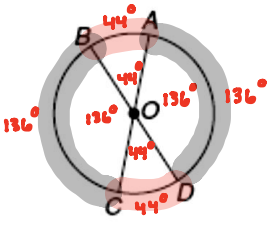
Find each measure.

1. $m\angle SCT = 75^\circ$
2. $m\angle SCU = 135^\circ$
3. $m\angle SCQ = 90^\circ$
4. $m\angle QCT = 165^\circ$



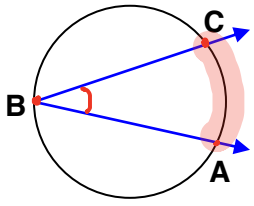
In ⊙O, $m\angle BOA = 44$. Find each measure.

5. $m\overset{\frown}{BA} = 44^\circ$
6. $m\overset{\frown}{BC} = 136^\circ$
7. $m\overset{\frown}{CD} = 44^\circ$
8. $m\overset{\frown}{ACB} = 316^\circ$
9. $m\overset{\frown}{BCD} = 180^\circ$
10. $m\overset{\frown}{AD} = 136^\circ$



Part 2: Inscribed Angles

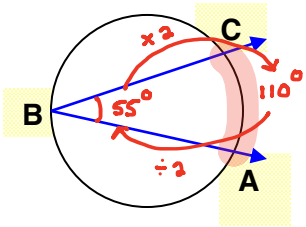
An inscribed angle is an angle whose vertex is ON the circle and whose sides contain chords of the circle.



$\angle CBA$ is an inscribed angle.

Minor arc CA is the intercepted arc of $\angle CBA$

The measure of an inscribed angle is half the measure of the intercepted arc.

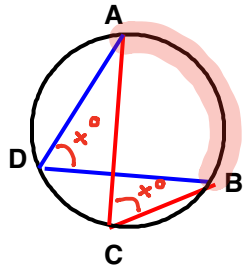


$$m\angle CBA = \frac{1}{2}m(arc\ CA)$$

For example: If the measure of arc CA is 110º, then $m\angle CBA = 55º$.

There are 3 corollaries that give us more information on the relationship between an inscribed angle and a circle.

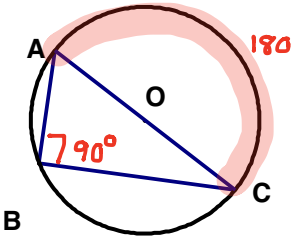
Corollary 1: Two inscribed angles that intercept the same arc are congruent.



$\angle ADB$ intercepts arc AB; $\angle ACB$ intercepts arc AB

Therefore, $\angle ADB \cong \angle ACB$

Corollary 2: An angle inscribed in a semicircle is a right angle.



If $\overline{AC}$ is a diameter of Circle O, then $\angle ABC$ is a right angle.

Corollary 3: The opposite angles of a quadrilateral inscribed in a circle are supplementary. (Remember an inscribed polygon has every vertex of the polygon touching the circle.)

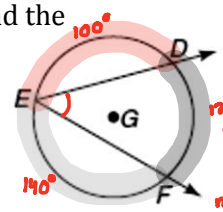
Add up to 180°

11. Use the given circle for both problems.

a. If the $m\angle DEF = 70$, find the measure of arc DF.

$m\overset{\frown}{DF} = 70 \times 2$

$m\overset{\frown}{DF} = 140°$



b. If the measure of arc DE = 100 and the measure of arc EF = 140, find the $m\angle DEF$

360 − (100 + 140)

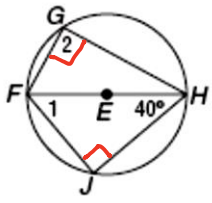
$m\overset{\frown}{DF} = 120°$

$m\angle DEF = \frac{120°}{2}$

$m\angle DEF = 60°$

12. Find the indicated angles.

$m\angle 1, m\angle 2$



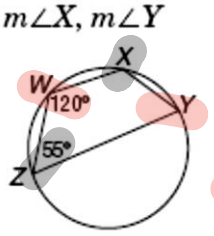
$m\angle 2 = 90°$

$m\angle 1 = 90 - 40°$

$m\angle 1 = 50°$

13. Find the indicated angles.

$m\angle X, m\angle Y$



$m\angle X = 180° - 55°$

$m\angle X = 125°$

$m\angle Y = 180° - 120°$

$m\angle Y = 60°$

Now, "You Try" these:

14. Refer to the figure. Find each measure.

a. $m\angle ABC = 90°$

b. $m\overset{\frown}{CD} = 118°$

c. $m\overset{\frown}{AD} = 62°$

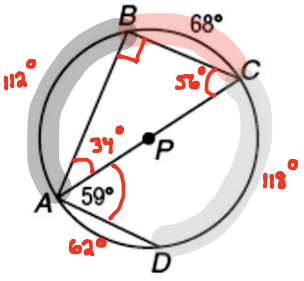
d. $m\angle BAC = 34°$

e. $m\angle BCA = 56°$

f. $m\overset{\frown}{AB} = 112°$

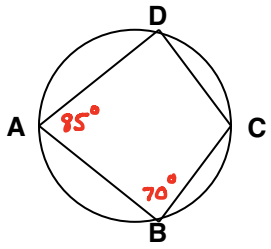
g. $m\overset{\frown}{BCD} = 186°$

h. $m\overset{\frown}{BDA} = 248°$



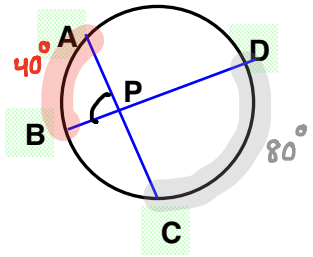
15. Find $m\angle D$ and $m\angle C$ if $m\angle A = 85$ and $m\angle B = 70$.

$m\angle C = 95^\circ$ $m\angle D = 110^\circ$



Part 3: Angles Formed Two Chords of a Circle (Vertex not at center of circle)

An angle formed by two chords of a circle whose vertex is not at the center of the circle has a measure equal to half the SUM of the intercepted arcs.



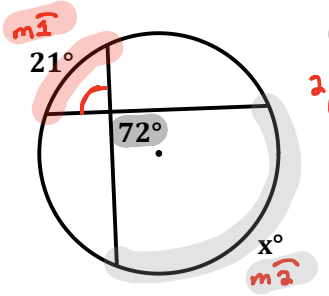
The $m\angle APB = \frac{1}{2}(m(arcAB) + m(arcDC))$

$m\angle APB = \frac{1}{2}(40 + 80)$

$= \frac{1}{2}(120) = 60^\circ$

For example: If the measure of arc AB is 40º and the measure of arc DC is 80°, the $m\angle APB = 60°$.

16) Find the value of *x*.



$m\angle = \frac{1}{2}(\quad + \quad)$

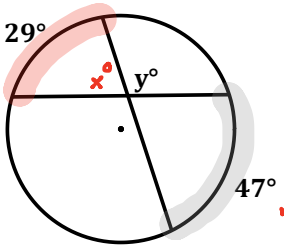
$2(72^\circ) = \frac{1}{2}(21^\circ + x)2$

$144^\circ = 21^\circ + x$

$-21 \quad -21$

$x = 123^\circ$

17) Find the value of *y*.



$m\angle x = \frac{1}{2}(29 + 47)$

$m\angle x = \frac{1}{2}(76)$

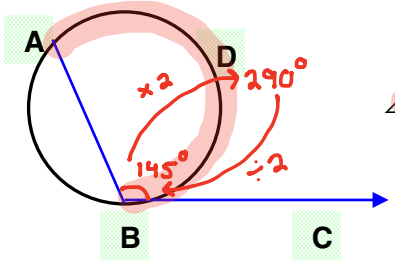
$m\angle x = 38^\circ$

$m\angle y = 180 - 38$

$m\angle y = 142^\circ$

Part 4: Angles Formed By Secants, Tangents and/or Chords

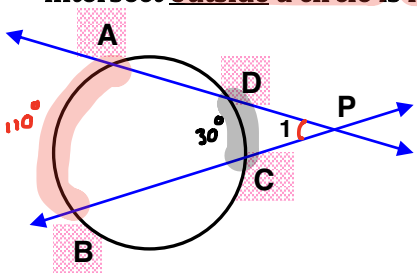
An angle formed by a tangent and a secant (or chord) that intersect on a circle at the point of tangency is one-half the measure of the intercepted arc.



$\angle ABC$ intercepts arc AB. The $m\angle ABC = \frac{1}{2} m(\text{arc } AB)$

For example: If the measure of arc ADB is 290º, then $m\angle ABC = 145º$.

The measure of an angle formed by two secants, a tangent and a secant, or two tangents that intersect outside a circle is half the DIFFERENCE of the intercepted arcs.



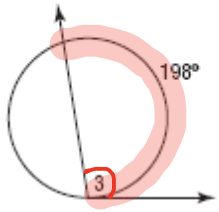
$$m\angle 1 = \frac{1}{2}(m(arcAB) - m(arcCD))$$

For example: If the measure of arc AB is 110 and the measure of arc CD is 30, then the

$$m\angle APB = \frac{1}{2}(110 - 30) = \frac{1}{2}(80) = 40^{\circ}.$$

Find the indicated angle measure

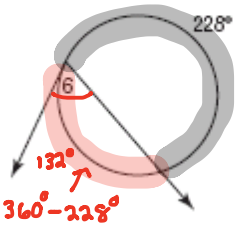
18. $m\angle 3$



$m\angle 3 = \frac{198^{\circ}}{2}$

$m\angle 3 = 99^{\circ}$

19. $m\angle 6$



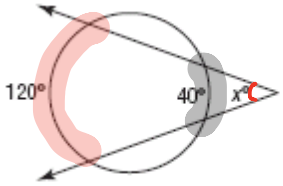
$360^{\circ} - 228^{\circ}$

$m\angle 6 = \frac{132}{2}$

$m\angle 6 = 66^{\circ}$

Find the value of x.

20.

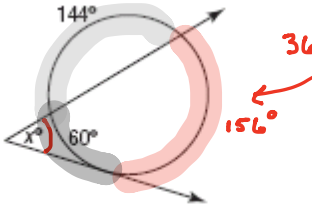


$m\angle x = \frac{1}{2}(120^{\circ} - 40^{\circ})$

$m\angle x = \frac{1}{2}(80^{\circ})$

$m\angle x = 40^{\circ}$

21.



$360 - (144 + 60)$

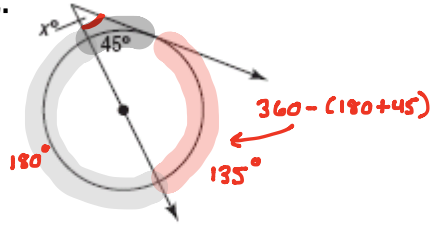
156°

$m\angle x = \frac{1}{2}(156 - 60)$

$m\angle x = \frac{1}{2}(96)$

$m\angle x = 48^{\circ}$

22.



$360 - (180 + 45)$

$m\angle x = \frac{1}{2}(135 - 45)$

$m\angle x = \frac{1}{2}(90)$

$m\angle x = 45^{\circ}$

23.